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# Plastic Analysis and Design of Steel Plate Shear Walls

Jeffrey Berman

*Department of Civil, Structural & Environmental Engineering, University at Buffalo*

***Research Supervisor: Michel Bruneau, Professor***

## Summary

*A revised procedure for the design of steel plate shear walls is proposed. In this procedure, the thickness of the infill plate is found using equations that are derived from plastic analysis of the strip model, which is widely accepted for the representation of steel plate shear walls. Equations are derived from a basic steel plate shear wall configuration but are shown to be conservative for more complex configurations. Fundamental plastic collapse mechanisms for several wall configurations are also given.*

## Introduction

Steel plate shear walls (SPSW) have sometimes been used as the lateral load resisting system in buildings. Until recently, the failure mode of SPSW was considered to be the out-of-plane buckling of the infill plates. This led engineers to design heavily stiffened plates that offered little economic advantage over reinforced concrete shear walls. However, as Basler (1961) demonstrated for plate girder webs, the post-buckling tension field action of SPSW can provide substantial strength, stiffness and ductility. The idea of utilizing the post-buckling strength of SPSW was first developed analytically by Thorburn et al., (1983) and verified experimentally by Timler and Kulak (1983). Studies performed to evaluate the strength, ductility, and hysteretic behavior of such SPSW designed with unstiffened infill plates have demonstrated their significant energy dissipation capabilities (Timler and Kulak 1983) and substantial economic advantages (Timler 1998).

At the time of this writing, there are no U.S. specifications or codes addressing the design of SPSW. The 2002 Canadian standard CAN/CSA S16-2002 (CSA 2002) now incorporates mandatory clauses for the design of SPSW. One of the models recommended to represent SPSW is the strip model developed by Thorburn et al., (1983). This model is generally recognized for providing reliable assessments of ultimate strength. In this study, the strip model is used as a basis to investigate the feasibility of plastic analysis as an alternative for the design of SPSW. Fundamental plastic collapse mechanisms are described for single story and multistory SPSW with either simple or rigid beam-to-column connections.

## Single Story Frames with Simple Beam-to-Column Connections

Consider the frame with inclined strips and simple beam-to-column connections shown in Figure 1. When the shear force, $V$, displaces the top beam by a value $\Delta$ sufficiently large to yield all the strips, the external work done is equal to $V\Delta$ (Figure 2).

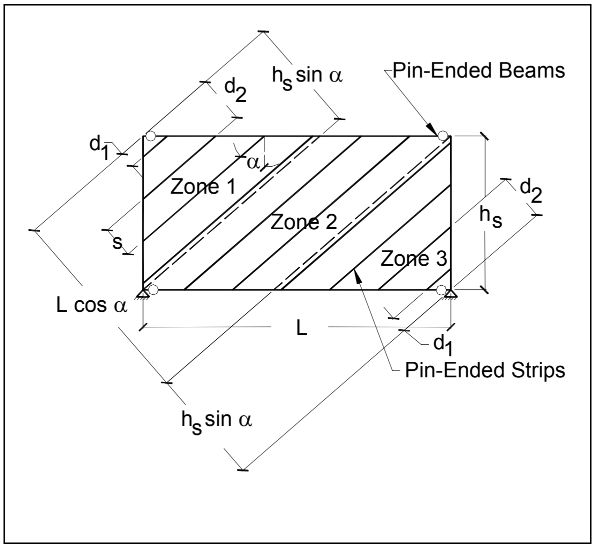

**Figure 1. Strip model of single story wall**

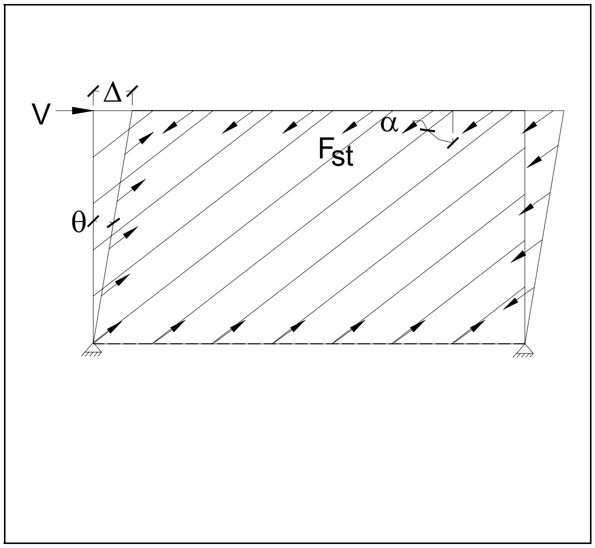

**Figure 2. Single story collapse mechanism**

If the beams and columns are assumed to remain elastic, their contribution to the internal work may be neglected when compared to the internal work done by the braces. Hence, the internal work is $[n_b A_{st} F_y \sin(\alpha)]\,\Delta$, where $n_b$ is the number of strips anchored to the top beam, $A_{st}$ is the strip area, $F_y$ is the strip yield stress, and $\alpha$ is the angle of inclination of the strips measured from vertical. This result can be obtained by the product of the yield force times the yield displacement of the strips, but for simplicity it can also be found using the horizontal and vertical components of these values. Note that the horizontal components of the yield forces of the strips on the columns cancel (the forces on the left column do negative internal work and the forces on the right column do positive internal work) and the vertical components of all the yield forces do no internal work because there is no vertical deflection. Therefore, the only internal work done is that due to the horizontal components of the strip yield forces anchored to the top beam. Equating the external and internal work gives:

$$V = n_b \; F_{st} \; \sin(\alpha) \tag{1}$$

Using the geometry shown in Figure 1, $n_b = [L \cos(\alpha)] / s$ and the strip force $F_{st}$ is again $F_y\, t\, s$. Substituting these into Eq. 1 and knowing that $(1/2) \sin(2\alpha) = \cos(\alpha) \sin(\alpha)$, the resulting base shear relationship is:

$$V = \frac{1}{2} F_y \; t \; L \sin(2\alpha) \tag{2}$$

## Single Story Frames with Rigid Beam-to-Column Connections

In single story SPSW having rigid beam-to-column connections (as opposed to simple connections), plastic hinges also need to form in the boundary frame to produce a collapse mechanism. The corresponding additional internal work is 4 $M_p\ \theta$, where $\theta = \Delta\ h_S$ is the story displacement over the story height and $M_p$ is the smaller of the plastic moment capacity of the beams ($M_{pb}$) or columns ($M_{pc}$). For most single-story frames that are wider than tall, if the beams have sufficient strength and stiffness to anchor the tension field, plastic hinges will typically form at the top and bottom of the columns and not in the beams. The ultimate strength of a single-story SPSW in a moment frame with plastic hinges in the columns becomes:

$$V = \frac{1}{2} F_y\ t\ L \sin\left(2\alpha\right) + \frac{4\ M_{pc}}{h_s} \tag{3}$$

In a design process, failure to account for the additional strength provided by the beams or columns results in larger plate thicknesses than necessary. In turn, larger thickness would translate into lower ductility demands in the walls and frame members. Therefore, neglect of additional strength provided by the beams or columns could be considered a conservative approach.

## Multistory Frames

For multistory SPSW with pin-ended beams, plastic analysis can also be used to predict the ultimate capacity. The purpose here is not to present closed-form solutions for all possible failure mechanisms, but to identify some key plastic mechanisms that should be considered in estimating the ultimate capacity of a SPSW. These could be used to define a desirable failure mode in a capacity design perspective and/or to prevent an undesirable failure mode, as well as to complement traditional design approaches.

In soft-story plastic mechanisms (Figure 3), the plastic hinges that would form in the columns at the mechanism level could be included in the plastic analysis. Calculating and equaling the internal and external work, the following general expression could be used for soft-story $i$ in which all flexural hinges develop in columns:

$$\sum_{j=i}^{n_s} V_j = \frac{1}{2} F_y\ t_i\ L \sin\left(2\alpha\right) + \frac{M_{pci}}{h_{si}} \tag{4}$$

where $V_j$ are the applied lateral forces above the soft-story $i$, $t_i$ is the plate thickness at the soft-story, $M_{pci}$ is the plastic moment capacity of the columns at the soft-story, $h_{si}$ is the height of the soft-story, and $n_s$ is the total number of stories. Note that only the applied lateral forces above the soft-story do external work and they all move the same distance ($\Delta$). The internal work is done only by the strips on the soft-story itself and by column hinges forming at the top and bottom of the soft-story. Using the above equation, the possibility of a soft-story mechanism should be checked at every story in which there is a significant change in plate thickness or column size. Additionally, the soft-story mechanism is independent of the beam connection type (simple or rigid) because hinges must form in the columns, not in the beams.

A second (and more desirable) possible collapse mechanism involves uniform yielding of the plates over every story (Figure 4). For this mechanism, each applied lateral force $V_i$ moves a distance

$\Delta_i = \theta\, h_i$, and does an external work equal to $V_i\, \theta\, h_i$, where $h_i$ is the elevation of the $i^{th}$ story. The internal work is done by the strips of each yielding story. It is important to note that the strip forces acting on the bottom of a story beam do positive internal work and the strip forces acting on top of the same beam do negative internal work. Therefore, the internal work at any story *i* is equal to the work done by strip yield forces along the bottom of the story beam minus the work done by strip yield forces on the top of the same beam. This indicates that in order for every plate at every story to contribute to the internal work, the plate thicknesses would have to vary at each story in direct proportion to the demands from the applied lateral forces. Even with this in mind, this mechanism provides insight into the capacity and failure mechanism of the wall. The general equation for the ultimate strength of a multistory SPSW with simple beam-to-column connections and this plastic mechanism (equating the internal and external work) is:

$$\sum_{i=1}^{n_s} V_i\, h_i = \sum_{i=1}^{n_s} \frac{1}{2} F_y \left(t_i - t_{i+1}\right) L\, h_i \sin\left(2\alpha\right) \qquad (5)$$

where $h_i$ is the $i^{th}$ story elevation, $n_s$ is the total number of stories, and $t_i$ is the thickness of the plate on the $i^{th}$ story.

After examining results of several different pushover analyses for such multistory SPSW, it has been observed that the actual failure mechanism is typically somewhere between a soft-story mechanism and uniform yielding of the plates on all stories. Finding the actual failure mechanism is difficult if hand calculations are performed. Therefore, a computerized pushover analysis should be used. However, the mechanisms described above will provide a rough estimate of the ultimate capacity. They will also provide some insight as to whether a soft story is likely to develop (by comparing the ultimate capacity found from the soft story mechanism with that of the uniform yielding mechanism).

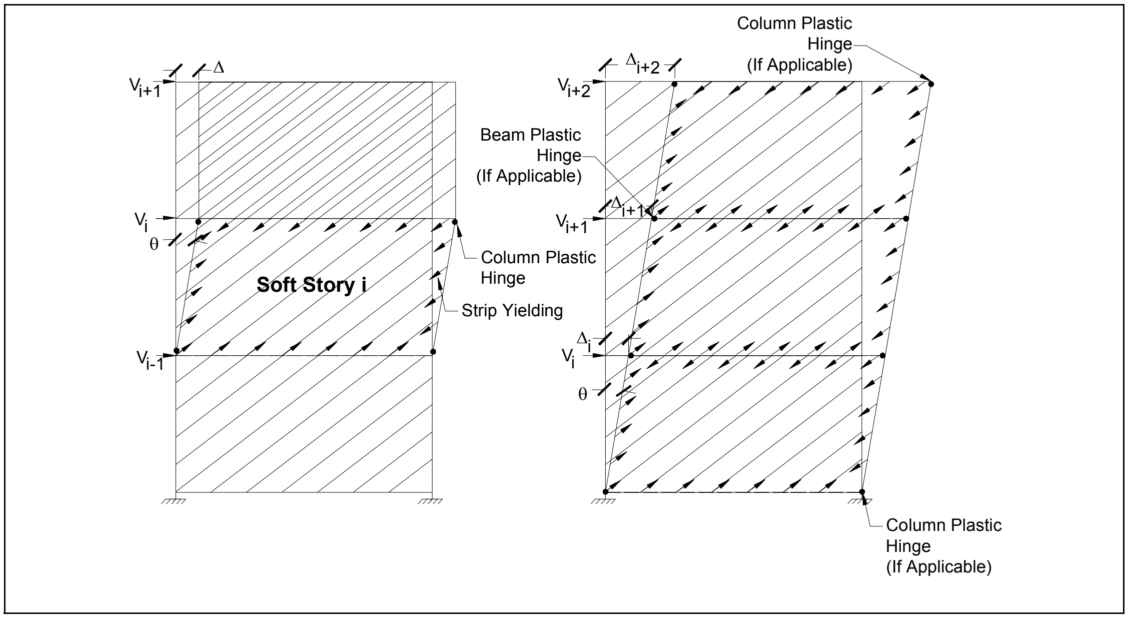

**Figure 3. Soft-story mechanism**

**Figure 4. Uniform yielding mechanism**

## Proposed Design Procedure

Using the results of the plastic analyses described previously, the infill plates of SPSW can be sized to consistently achieve the desired ultimate strength. The procedure is simple, even for a multistory SPSW, and neglecting the contribution of plastic hinges in beams and columns will always give a conservative design in the case of rigid beam-to-column connections. The proposed procedure requires the designer to:

1. Calculate the design base shear, and distribute it along the height of the building as described by the applicable building code;

2. Use the following equation to calculate the minimum plate thicknesses required for each story:

$$t = \frac{2\, V_s\, \Omega_s}{F_y\; L \sin(2\alpha)} \tag{6}$$

   where $\Omega_s$ is the system overstrength factor described in FEMA 369 (BSSC 2001) and $V_s$ is the design story shear found using the equivalent lateral force method;

3. Develop the strip model for computer (elastic) analysis using the equation from Timler et al., (1983) to calculate the angle of inclination of the strips;

4. Design beams and columns according to capacity design principles (to insure the utmost ductility) or other rational methods using plate thicknesses specified (in case these thicknesses exceed the minimum required for practical reasons);

5. Check story drifts against allowable values from the applicable building code;

## Conclusions

Plastic collapse mechanisms for single and multistory SPSW with simple and rigid beam-to-column connections have been investigated and simple equations that capture the ultimate strength of SPSW have been developed. Using the results of these plastic analyses, a new procedure for the sizing of the infill plates has been proposed. The proposed procedure allows the engineer to control the ultimate failure mechanism of the SPSW, and directly accounts for structural overstrength.

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