Advanced Time-Frequency Analysis Applications in Earthquake Engineering

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Summary
Future design procedures for civil structures, especially those to be protected from extreme and blast related loads, will need to account for temporal evolution of their frequency content. Separate time analysis and frequency analysis by themselves do not fully describe the nature of these nonstationary dynamic loads. In the past few years, significant effort has been devoted to wavelets and time-frequency analysis. This article attempts to briefly present certain techniques which are currently available, and suggests possible applications in the area of earthquake engineering. Further, it gives an outline of current research topics, some preliminary results on earthquake signal representation using time-frequency analysis techniques, and possible future research directions in this area. To comply with the space limitations of this article, only references that are readily available in the form of books are cited.

Introduction
Spectral analysis using the Fourier Transform has been one of the most important and most widely used tools in earthquake engineering. Over the past few years, however, researchers have become aware of the limitations of this technique, especially in the case of nonstationary signals, and of nonlinear systems.

Monte Carlo simulations are often used in the design of a structure subjected to earthquake excitations. These simulations require the generation of artificial earthquake signals, compatible with a design power spectrum, for input into the structural system. Further, the techniques used for the generation of the artificial earthquake signals must account for their inherent nonstationary frequency characteristics. Also, capturing evolutionary and localized features of the response of linear and nonlinear dynamic systems subject to nonstationary inputs is not feasible within the limits of traditional spectral analysis. It is clear that other techniques need to be developed.

In this direction, significant effort has been devoted in the development of Time-Frequency (TF) methods, which allow a temporal representation of the spectral characteristics of the signal. Such TF methods involve the short-time Fourier transform (STFT), the wavelet transform (WT), the Wigner-Ville distribution (WVD), the best basis search algorithm using wavelet packets or Malvar-Wilson wavelets, and the matching pursuit (MP) algorithm using Gabor atoms or chirplets (Hubbard 1998).
**Fourier Analysis and Wavelets**

Fourier analysis is based on the notion that any regular periodic function and certain nonperiodic functions with finite integral can be expressed as a sum of trigonometric functions in an infinite time framework (Boggess and Narcowich, 2001). Fourier transform gives a unique representation of the signal in the frequency domain and provides information about which frequencies appear in the signal but not about the time instants in which these frequencies are encountered. The time information is not lost through the transform but it is hidden in the phases. Thus, it is able to yield a perfect and unique reconstruction of the signal. There are many signals in nature for which the Fourier transform is not just a mathematical artifact but whose frequencies correspond to actual physical waves that make the signal. The physical interpretation of the signal is then simple. However, there are cases for which the Fourier analysis does not provide results that can be physically interpreted. As a result of Whitacker’s sampling theorem of 1935, its subsequent application in communication theory by Shannon in 1949, and the discrete fast Fourier transform algorithm by Cooley and Tukey in 1965, the improved computational efficiency of the Fourier analysis has made it the most widely used mathematical tool in a vast field of applications. However, Fourier analysis and regular power spectra representations are not suitable for all kinds of problems and signals. The natural phenomena are usually nonlinear and the majority of the signals have changing frequency contents.

For the aforementioned reasons, other methods that account for joint time-frequency representation of the signals have been developed. The most widely used one is the short-time Fourier transform (STFT). The basic concept in this method is to divide the signal into small segments (windows) of the same width and perform Fourier analysis on each of them to get the frequencies present in each segment (Cohen 1995). If good localization in time is desired, then a narrow window in the time domain has to be chosen. However, if good frequency localization is desired, a narrow window in the frequency domain has to be chosen. Thus, there is a trade-off between time and frequency localization governed by Heisenberg’s uncertainty principle. Related to the STFT is the Gabor transform. In 1946, Gabor gave a new perspective of the STFT by introducing a tiling of the time-frequency domain as shown in Figure 1c. Gabor expansion is the best way to compute the inverse discrete STFT (Qian and Chen, 1996).

![Figure 1. Time and frequency domain representations and tiling](image_url)
Wavelets are a natural extension of the Fourier analysis (Daubechies 1992). A wavelet is small wave whose energy is concentrated in time (Figure 2). In order to detect the characteristics of a signal, we compare it to a given elementary function. When the scaled and time-shifted elementary functions are used for this purpose, the resulting representation is called wavelet analysis and the elementary function is known as a mother wavelet. From another perspective, instead of having a constant window as in the case of STFT, wavelet analysis considers variable size windows, allowing for the use of long time segments for capturing the low frequency contents, and narrow time segments for capturing high frequency contents (Figure 1d). Although wavelet analysis is a time-scale analysis technique, connection to the time-frequency analysis can be made. Low scales correspond to compressed wavelets which are capable of capturing rapidly changing features of the signal linked to high frequencies. On the other hand, high scales correspond to dilated wavelets that are able to capture the slowly changing features of the signal linked to low frequencies. As in the case of Fourier analysis, we have continuous and discrete transforms. The discrete transforms can be redundant, orthogonal, or biorthogonal. There are infinitely many wavelets in the sense that any function concentrated in time can serve as an analyzing function. A plethora of wavelets has been developed to best suit several problems in science and engineering related to transient, time-variant, or nonstationary phenomena. This gives the method a great flexibility. However, this can also be a disadvantage since one has to choose the best wavelet for the application in hand. For instance, although wavelets have been successfully used in the solution of nonlinear differential equations, they have not provided a straightforward method for the solution of differential equations the way the Fourier analysis has. Wavelets have become a common language for people in different fields that had been using the same techniques under different names. This has brought a revolution in the field of time-frequency analysis. A great amount of literature has been developed on wavelets in the past few years, both on the mathematical foundation of the method and on their applications in numerous fields.

The STFT and WT are based on the concept of finding the similarity between the signal and the analyzing functions and have the disadvantage that Heisenberg’s uncertainty principle restrains their time-frequency resolution. Another approach, the Wigner-Ville distribution (WVD), based on time-frequency density function, yields better time-frequency resolutions. The WVD can be used to derive the instantaneous frequency function and the spectrogram. Disadvantages of the WVD include the cross-terms that affect the time-frequency resolution, and the negative values that the distribution can take (Cohen 1995).
**Best Basis and Matching Pursuit Algorithms**

Both Fourier and wavelet analysis have limitations. Fourier analysis gives good results for regular periodic signals (Figure 3a) and wavelet analysis is suitable for highly nonstationary signals that possess sudden picks and discontinuities (Figure 3c). Other approaches have been examined, and several algorithms and analyzing functions have been proposed (Jaffard et al., 2001). These include the best basis, and the matching pursuit algorithms. The best basis search algorithm uses wavelet packets, Malvar-Wilson wavelets, or generalized Malvar-Wilson wavelets. The matching pursuit (MP) algorithm uses Gabor atoms, or chirplets.

![Fourier Analysis](a) Fourier Analysis ![Time-Frequency Analysis](b) Time-Frequency Analysis ![Wavelet Analysis](c) Wavelet Analysis

Figure 3. Time-Frequency plane representations and suitable types of analysis.

In the first approach, the signal is expressed as a linear combination of time-frequency atoms. The atoms are obtained by dilations of the analyzing functions, and are organized into dictionaries as wavelet packets, or Malvar-Wilson wavelets. Wavelet packet atoms are waves indexed by time, scale, and frequency (Figure 4a). For any orthogonal analyzing function, it is possible to generate a dictionary of wavelet packet bases. The Malvar-Wilson wavelets are functions which have the form of Figure 4b. They are characterized by an attack period, a stationary period, and a decay period whose duration can be arbitrarily and independently chosen. A modified version of the Malvar-Wilson wavelets that takes into account a linear modulation of the frequencies can be found in Jaffard et al., 2001.

The best basis algorithm described in Wickerhauser (1994) uses a minimum entropy criterion and gives the most concise description for a signal for the dictionary in hand. The representation of the signal depends on the size of the dictionary, thus leading to large dictionaries and high computational cost to account for more kinds of signals and achieve high time-frequency resolution. The application of the best basis search for the wavelet packet dictionary is equivalent to an optimal filtering of the signal, whereas, the Malvar-Wilson wavelets dictionary is equivalent to an optimal segmentation of the signal. For any given signal, the best basis algorithm decides which base represents the signal more efficiently.
A chirplet function is shown in Figure 5 along with its WVD. Chirplets have a short smooth Gaussian envelope and a linear frequency modulation. They are characterized by four parameters which allow for localization and modulation: the time center, the frequency center, the variance, and the frequency change rate. Since they are derived from the Gaussian function, they always have a nonnegative WVD. A signal can be adaptively expanded in terms of chirplet atoms using the matching pursuit algorithm and the adaptive spectrogram that can be derived by taking the WVD of the signal. The Gabor atoms can be derived from the chirplets by setting the frequency change rate to zero. Chirplets and Gabor atoms do not form bases and their dictionaries are redundant. The Matching Pursuit algorithm allows for the decomposition of the signals into elementary functions that do not form bases. It is a basic component of the adaptive Gabor expansion, and the adaptive chirplet transform; a detailed description of the algorithm can be found in (Qian 2001).

Current Research and Future Work
The adaptive Gabor expansion and the adaptive chirplet transform have been implemented to obtain representations of earthquake records. Preliminary results show that, in terms of signal
expansion, the two methods give almost the same results. In this regard, the level of sophistication of the analyzing function used to capture the physics of the seismic records is examined. Comparisons with other methods of analysis such as wavelet analysis using harmonic wavelets (Figure 2), and classic Fourier analysis have been conducted. As expected, the adaptive methods give better results.

Further, effort is devoted in the direction of transition from individual spectrograms, derived by the WVD of the expanded signal, to evolutionary power spectrum. The spatial variation of earthquake records will be of interest as well. Thus, the extension to the multivariate case, which inevitably leads to the concept of the cross-power spectrum, is attempted and efficient expressions for the evolutionary coherency functions are pursued. In addition, a methodology allowing for the simulation of artificial, nonstationary signals, such as earthquakes, and other extreme loads (blast, wind, ocean waves, etc.) based on these evolutionary power spectra, is being developed.

Finally, capturing of the evolutionary and localized features of the response of linear and nonlinear dynamic systems subject to nonstationary inputs will also be pursued, especially in conjunction with the design of critical structures which may be exposed to extreme, low probability loads. Advanced time-frequency analysis techniques are used to observe the shifting of the natural frequencies of nonlinear structures and the changes on modal damping. It is clear that these techniques can become useful in health monitoring and structural control.

**Concluding Remarks**

The classic spectral analysis in the frequency domain has been discussed, and several limitations that derive from the use of Fourier analysis have been pointed out. Also, the alternative of employing short-time Fourier transform has been considered. Then, modern techniques of time-frequency analysis have been discussed. In this regard, wavelet transform has been presented. A plethora of applications of wavelet transform in earthquake engineering is available in the literature. Several issues regarding the interpretation of the results, since wavelet transform is a time-scale transform, need further attention. Alternatives to the wavelets-based scheme have been presented. Applications of these techniques to earthquake engineering include the derivation of evolutionary power spectra of dynamic loads, and the capturing of the changing frequency content of the response. In this context, it is believed that stochastic dynamics and time-frequency analysis merit additional attention.

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**References**


