Force Based Design Fundamentals

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Learning Outcomes

- Explain difference between elastic forces, actual forces and Modified Design Forces
- Define a plastic hinge
- Explain Capacity Protection Philosophy
- Describe the Component Capacity/Demand Retrofit Method (Method C)
- Describe how P-Δ effects are considered in design
Force vs. Displacement

Elastic Analysis & Response

Displacement - $\Delta$

$F_{EQ}$

$\Delta_{EQ}$

Topic Applicability

<table>
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<th>Force</th>
<th>Displ.</th>
<th>Retro</th>
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**Force vs. Displacement**

- For moderate-large earthquakes, $F_{EQ}$ is a very large force (can be $> structure weight) and it is uneconomical to design an *ordinary* bridge to resist this force elastically.
- Inelastic behavior (damage) is therefore accepted, provided it:
  - does not cause collapse
  - is ductile in nature (not brittle), and
  - occurs in designated components (members)
- Exceptions are made for *special* structures where the higher cost of elastic (or almost elastic) behavior can be justified.
First Order Rigid-Plastic analysis assumes all deformations take place at discrete regions, called plastic hinges, and that a sufficient number of plastic hinges have formed to form a mechanism in the pier/bent.
Force vs. Displacement

Idealized Elasto-Plastic Response

Displacement - $\Delta$

Lateral Force

First-Order Rigid-Plastic Response

Elastic Response

Idealized Elasto-Plastic Response
**Force vs. Displacement**

Displacement - \( \Delta \)

**Actual Response Milestones**

1. Pseudo-yield Point
2. Maximum Plastic Deformations
3. Onset of Collapse
4. Collapse
Force vs. Displacement

Elastic vs. Actual Response
Force vs. Displacement

Idealized Elasto-Plastic Response

First-Order Rigid-Plastic Response

Elastic Response

Actual Response

Idealized Elasto-Plastic Response

Lateral Force

Displacement - $\Delta$

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Seismic Technologies for Extreme Loads
Force vs. Displacement

- Elastic Analysis conducted using linear methods.
- First-order elasto-plastic behavior includes reduced pier/bent stiffness and strength with progressive plastic hinge formation.
- Second-order elasto-plastic behavior traces formation of plastic hinges and includes geometric non-linearities (e.g. P-Δ).

**Pier Response with Higher Order Effects**

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**Force vs. Displacement**

**Member strengths:**

- *Nominal strength, $S_n$*
- *Design strength, $S_d = \phi S_n$*
  where $\phi$ is the strength reduction factor $< 1.0$
  (see AASHTO LRFD Specifications)
- *[Expected strength, $S_e = \phi_e S_n$]*
  where, in absence of mill certificates,
  $\phi_e = 1.2$ for steel and $1.3$ for concrete members]
- *Over-strength, $S_o = \phi_o S_n$*
  where $\phi_o$ is the over-strength factor $> 1.0$  ($1.7 – 2.7$)
F_{overstrength} = \phi_o F_{mechanism} \quad \text{where } \phi_o = \text{over-strength factor}

\text{Over-strength Forces}
Force vs. Displacement

\[ \mu = \frac{\Delta_{\text{max}}}{\Delta_{\text{yield}}} \]

Displacement - \( \Delta \)

Displacement Ductility Factor - \( \mu \)
Force vs. Displacement

- To calculate $\Delta_{\text{max}}$
  - Nonlinear time history analysis using software such as SAP2000, SEISAB-NL, ADINA, ABAQUS…
  - Elastic analysis using either ULM, SMSA, MMSA, TH to find maximum elastic displacement $\Delta_{\text{EQ}}$ and then assume
    - either: Equal displacements, i.e. $\Delta_{\text{max}} = \Delta_{\text{EQ}}$
    - or: Equal work done during elasto-plastic response as during elastic response, and solve for $\Delta_{\text{max}}$
Define \( R = \frac{F_{\text{elastic}}}{F_{\text{mechanism}}} \)

(Respond Modification Factor)

Then: \( \Delta_{\text{max}} = \Delta_{\text{EQ}} \)

and: \( \mu = \frac{\Delta_{\text{max}}}{\Delta_{\text{yield}}} = R \)

(Displacement Ductility Factor)

Relationship between ductility and “R”

Equal Displacement Assumption
Define $R = \frac{F_{\text{elastic}}}{F_{\text{mechanism}}}$ (Response Modification Factor)

Then: $\Delta_{\text{max}} = \Delta_{\text{EQ}}$

and: $\mu = \frac{\Delta_{\text{max}}}{\Delta_{\text{yield}}} = R$

(Displacement Ductility Factor)

**Relationship between ductility and “R”**

*Equal Displacement Assumption*
Define $R = \frac{F_{\text{elastic}}}{F_{\text{mechanism}}}$

(Response Modification Factor)

Then:

$\Delta_{\text{max}} = \frac{(R \Delta_{\text{EQ}} + \Delta_{\text{yield}})}{2}$

and:

$\mu = \frac{\Delta_{\text{max}}}{\Delta_{\text{yield}}} \approx \frac{(R^2 + 1)}{2}$

(Displacement Ductility Factor)

Relationship between ductility and "$R$"

Equal Work Done Assumption

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**Force vs. Displacement**

Define $R = \frac{F_{\text{elastic}}}{F_{\text{mechanism}}}$ (Response Modification Factor)

Then:

$\Delta_{\text{max}} = \frac{(R \Delta_{\text{EQ}} + \Delta_{\text{yield}})}{2}$

and:

$\mu = \frac{\Delta_{\text{max}}}{\Delta_{\text{yield}}} \approx \frac{(R^2 + 1)}{2}$

(Displacement Ductility Factor)

**Relationship between ductility and “R”**

*Equal Work Done Assumption*

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Seismic Technologies for Extreme Loads
Force vs. Displacement

\[ \mu = \frac{\Delta_{\text{max}}}{\Delta_{\text{yield}}} \approx R \]

\[ \mu = R = 1 \]
\[ \mu = R = 2 \]
\[ \mu = R = 3 \]
\[ \mu = R = 5 \]

Displacement - \( \Delta \)

Increasing Ductility

Substructure Type

Strength and Ductility Relationship

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**Force vs. Displacement**

\[ \mu = \frac{\Delta_{\text{max}}}{\Delta_{\text{yield}}} \approx R \]

- \( \mu = R = 1 \)
- \( \mu = R = 2 \)
- \( \mu = R = 3 \)
- \( \mu = R = 5 \)

**Increasing Ductility**

**Substructure Type**

**Strength and Ductility Relationship**

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Seismic Technologies for Extreme Loads
Force vs. Displacement

\[ \mu = \frac{\Delta_{\text{max}}}{\Delta_{\text{yield}}} \approx R \]

Displacement - \( \Delta \)

Strength and Ductility Relationship

NOTE: Increasing flexibility longer period, higher \( \Delta_{EQ} \)

Substructure Type

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**Force vs. Displacement**

- **Lateral Force vs. Displacement - Δ**
  - $F_{\text{elastic}}$
  - $F_{\text{overstrength}}$
  - $F_{\text{mechanism}} = F_{\text{design}}$
  - $F_{\text{elastic}}/R$

- **Axial Force**
  - $(P_n, M_n)$ w/ Over-strength
  - $\Phi P_n \Phi M_n$
  - $P, M_{\text{design}}$
  - $P, M_{\text{elastic}}$

- **Elastic vs. Modified Design vs. Over-strength Forces**

**Elastic vs. Modified Design vs. Over-strength Forces**

- $F_{\text{elastic}}$ & $P, M_{\text{elastic}}$
  - Force effect generated from Extreme Event 1 load combination

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Seismic Technologies for Extreme Loads
The principle of *Capacity Protection Design* is that yielding in one component (or member) will cap (or limit) the forces and moments in an adjacent component (member). i.e. reaching the elastic capacity (yield) of one member protects adjacent members from excessive forces.
Capacity Protection

Bridge Geometry

F, Δ

A B C D

E F

h₁ h₂

Topic Applicability

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Seismic Technologies for Extreme Loads
Capacity Protection

Bridge Geometry

Bending Moment Diagram

Two plastic hinges in BE,
\[ V_{EB, \text{max}} = 2 \frac{M_p}{h_1} \]
Capacity Protection

Bridge Geometry

Bending Moment Diagram

Two plastic hinges in BE,
\( V_{EB} \) max = 2 \( M_p \) / \( h_1 \)

\[ F = V_{EB} + V_{FC} \]

The diagram shows the force and displacement for a bridge, with a bending moment diagram indicating key points such as 1, 2, 3, and 4. The equation for the force balance is given as:

\[ F = V_{EB} + V_{FC} \]
Two plastic hinges in EB,

\[ V_{EB_{\text{max}}} = \frac{2 M_p}{h_1} \]

Maximum shear and moment in foundation at E is limited by yielding in EB, i.e. the foundation is *capacity protected* by column yield.
Capacity Protection

Bridge Geometry

Bending Moment Diagram

Two plastic hinges in EB,
\[ V_{EB}^{\text{max}} = \frac{2M_p}{h_1} \]

NOTE: For design \( M_p = \phi M_n \), but to calculate max. value of \( V_{EB} \), should use the overstrength moment, i.e. \( M_p = \phi_o M_n \) where \( \phi_o > 1 \)
Seismic Retrofit Process

Three step process:
- Screening and Prioritization
- Detailed Evaluation
- Retrofit – strategies, approaches and measures
Evaluation Methods

- FHWA Manual describes 5 basic methods for evaluation
- Methods vary in rigor from ‘no-analysis’ to ‘nonlinear dynamic time history analysis’
- Selection / application depends on the Seismic Retrofit Category A – D for the bridge (seismic hazard and performance required), and its geometry (regularity).
Evaluation Methods

- A and B: Default capacity method checks capacity of components due to non-seismic loads against specified minima – no analysis is required (e.g. connections and support lengths)
- C: Capacity/demand method compares component capacities against force demands from an elastic analysis - on a component-by-component basis
Evaluation Methods

D: Capacity/spectrum method(s) use nonlinear capacity models for individual piers (or complete bridge), i.e. a pushover curve, and displacement demands from an elastic analysis.

E: Nonlinear dynamic method explicitly models nonlinear behavior of components in a time history analysis of complete bridge.
Evaluation Method C

- Capacity / Demand Ratio Method
  - *Force* demands are calculated by elastic methods, such as multi-modal spectral analysis method (i.e. without regard to any yielding that may occur). Elastic *displacements* also used.
  - Capacities are obtained from combination of theory (strength of materials) and engineering judgment, for each major component.
  - Gives good results for bridges that behave elastically, or nearly so.
Evaluation Method C

Five step procedure (FHWA 2006):
1. Determine applicability of the method
2. Determine capacity, $Q_{ci}$ for all components $(i)$ using theory and engineering judgement (e.g. Appendix D, FHWA Manual, 2006)
3. Determine sum of non-seismic force and displacement demands, $\Sigma Q_{NSi}$ for all components for each load combination in LRFD design specification
Evaluation Method C

Procedure continued:

4. Determine seismic demand, $Q_{EQi}$ on each component by an elastic method of analysis.

5. For each component determine capacity/demand ratio from:

$$r_i = \frac{Q_{ci} - \sum Q_{NSi}}{Q_{EQi}}$$
Evaluation Method C

- If $r_i > 1.0$ component has adequate capacity

- If $0.5 \leq r_i < 1.0$ component may be acceptable without retrofitting, depending on other deficiencies in member, if any, and consequences of failure

- If $0.5 > r_i$ component has inadequate capacity and retrofitting is indicated
Evaluation Method C

- Note that for each component, several c/d ratios ($r_i$) may need to be calculated.
- **Example 1:** Support lengths and bearings

\[
\begin{align*}
    r_{bd} &= \frac{\Delta_s (c) - \Delta (d)}{\Delta_{eq} (d)} \\
    r_{bf} &= \frac{V_b (c)}{V_b (d)}
\end{align*}
\]
Evaluation Method C

- **Example 2: Columns**

Five (5) c/d ratios should be found for each column for the following:

- Anchorage length of longitudinal rebar, $r_{ca}$
- Transverse confinement, $r_{cc}$
- Splice length, $r_{cs}$
- Shear force, $r_{cv}$
- Bending moment, $r_{ec}$
Equilibrium in deformed state requires $\sum M_A = 0$, i.e. $F h + P \Delta = (K_{\text{pier}} \Delta) h$

Therefore $F = (K_{\text{pier}} - P / h) \Delta = K'_{\text{pier}} \Delta$
**P-Δ Effects in Elastic Bridge Columns**

Elastic capacity curve for column, slope = $K_{pier}$

Elastic capacity curve with P-Δ included, slope = $K'_{pier}$

Loss in capacity at displacement $\Delta$, due to P-Δ effect

$= \frac{P\Delta}{h}$

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**Topic Applicability**

- Force: ✅
- Displ.: ✅
- Retro: ✅
**P-Δ Effects in Yielding Bridge Columns**

Elasto-plastic capacity curve for column:
- initial slope = $K_{pier}$
- second slope = 0

Elasto-plastic capacity curve with P-Δ included:
- initial slope = $K'_{pier}$
- second slope = $-\frac{P}{h}$

Loss in capacity at displacement $\Delta$ due to P-Δ effect:
- $= \frac{P\Delta}{h}$
P-Δ Effects in Yielding Bridge Columns

Elasto-plastic capacity curve for column; initial slope = \( K_{\text{pier}} \), second slope = 0

Elasto-plastic capacity curve with P-Δ included; initial slope = \( K'_{\text{pier}} \), second slope = \(-\frac{P}{h}\)

Loss in capacity at displacement \( \Delta \) due to P-Δ effect

\[ = \frac{P\Delta}{h} \]
**P-Δ Effects in Yielding Bridge Columns**

- AASHTO LRFD Specifications and FHWA Retrofit Manual require that if $\frac{P\Delta}{h} > 0.25 \text{ Mp} / \text{h}$
  - a refined analysis must be undertaken that explicitly includes nonlinear geometric effects ($P-\Delta$)
- Encouraged to limit $\Delta$ such that $\Delta_{\text{max}} < 0.25 \text{ Mp} / P$
Learning Outcomes

- Explain difference between elastic forces, actual forces and Modified Design Forces
- Define a plastic hinge
- Explain Capacity Protection Philosophy
- Describe the Component Capacity/Demand Retrofit Method (Method C)
- Describe how P-Δ effects are considered in design
Thank you...