Evaluation of Members, Connections, and Subsystems

Tim Ingham

Outline

- Member capacity
  - Axial
  - Flexural
  - Ductility

- Connection capacity
  - Riveted
  - Gusset plates
  - Net section

- Displacement capacity of subsystems

- Capacity demand analysis
Axial Capacity, per AASHTO

**Tension**

\[ P_n = f_y A_g \]
\[ P_n = f_u A_g U \]

Where

- \( f_y \) = yield strength of steel
- \( f_u \) = tensile strength of steel
- \( A_g \) = gross area of section
- \( A_n \) = net area of section
- \( U \) = reduction factor for shear lag

**Compression**

If \( \lambda \leq 2.25 \) then

\[ P_n = 0.66 \frac{f_y A_g}{\lambda} \]

If \( \lambda > 2.25 \) then

\[ P_n = \frac{0.88 f_y A_g}{\lambda} \]

Where

\[ \lambda = \left( \frac{K l}{r^2} \right) \frac{f_y}{E} \]

Where

- \( K \) = the effective length factor
- \( l \) = the unbraced length of the member
- \( r \) = the radius of gyration
- \( E \) = the elastic modulus
Axial Capacity, Slender Cross-Section

A cross-section is slender if $\lambda > \lambda_r$ for one of its components. The component will buckle before reaching the yield strain of the material.

<table>
<thead>
<tr>
<th>Member</th>
<th>Ratio</th>
<th>$\lambda_7$</th>
<th>$\lambda_9$</th>
<th>$\lambda_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flanges of I-shapes</td>
<td>$\frac{b}{t}$</td>
<td>$\frac{141}{\sqrt{F_y - 10}}$</td>
<td>$\frac{65}{\sqrt{F_y}}$</td>
<td>$\frac{52}{\sqrt{F_y}}$</td>
</tr>
<tr>
<td>Flanges of boxes</td>
<td>$\frac{b}{t}$</td>
<td>$\frac{238}{\sqrt{F_y - F_r}}$</td>
<td>$\frac{190}{\sqrt{F_y}}$</td>
<td>$\frac{110}{\sqrt{F_y}}$</td>
</tr>
</tbody>
</table>

Follow the procedure described in Appendix B of the Load and Resistance Factor Design Specification for Structural Steel Buildings
For Un-Stiffened Elements

\[ Q_s = 1.415 - 0.00437 \frac{b}{t} \sqrt{f_y} \quad \text{if} \quad \frac{95.0}{f_y} < \frac{b}{t} < \frac{176}{f_y} \]

\[ Q_s = \frac{20000}{f_y \left(\frac{b}{t}\right)^2} \quad \text{if} \quad \frac{176}{f_y} \leq \frac{b}{t} \]

where
- \( b \) = the width of the leg
- \( t \) = the thickness of the leg

For Stiffened Elements

\[ Q_s(f) = \frac{b_e(f)}{b} = \frac{326}{b} \left( 1 - \frac{64.9}{b \sqrt{f}} \right) \]

with
- \( b_e(f) \) = the effective width
- \( b \) = the actual width
- \( f \) = the stress in the element
Axial Capacity, Slender Cross-Section

\[ Q_a(f) = \frac{\sum b_t(f)t}{\sum b_t} \]

\[ Q = Q_a Q \]

The nominal compression capacity is

If \( Q\lambda \leq 2.25 \) then \( P_n = Q 0.66^{Q\lambda} f_y A_g \)

If \( Q\lambda > 2.25 \) then \( P_n = \frac{0.88 f_y A_g}{\lambda} \)

the first equation requires an iterative solution

\[ f_{cr} = Q(f_{cr}) 0.66^{Q(f_{cr})\lambda} f_y \]

Example: Slender Cross-Section

[Diagram of slender cross-section]
Splices in Built-Up Members

- Gross section capacity
- Net section capacity
  - Of splice plates
  - Of member
  - Could occur through intermediate plates
- Load transfer capacity, through rivets or bolts
- If member ductility demands exceed unity, then splices should be 25% stronger than the member
Laced Members

- Not treated in modern codes

- Tensile capacity
  - Use usual methodology
  - Don't count the laces in gross area

- Compressive capacity
  - Use usual methodology
  - Member stability
  - Section stability, based on width-to-thickness ratios
    - With a modification
  - Consider also the stability of the member between the points of attachment of the laces
Compressive Capacity of Laced Members

For \( \frac{KL}{r} > 40 \), \( K' = K \sqrt{1 + \frac{300}{(KL/r)^2}} \)

For \( \frac{KL}{r} \leq 40 \), \( K' = 1.1K \)

where

- \( K \) = the effective length factor
- \( l \) = the unbraced length of the member
- \( r \) = the radius of gyration

Proportions of Laced Members

\[ \frac{a}{r_{\text{min}}} \leq \min \left\{ 40, \frac{2KL}{3r} \right\} \]

Per the AASHTO Standard Specifications, the slenderness of longitudinal elements must satisfy
Ill-Proportioned Laced Member

- Must consider interaction of local and global buckling

\[
\frac{a}{r_{\text{min}}} > \min \{40, \frac{2KL}{r}\}
\]
Actual Member

With Partial Lacing

No convergence beyond this point
Shear Strength of Laced Members

For a single plane of single lacing

\[ V_1 = \frac{T}{\sqrt{L^2 + T^2}} P_n\left(\sqrt{L^2 + T^2}\right) \]

where

\( P_n(l) \) is the compressive capacity of a lacing bar and implicitly considers the bar’s properties.

Check for external shears plus 2% of compressive capacity.

For a single plane of double lacing

\[ V_1 = \frac{2T}{\sqrt{L^2 + T^2}} P_n\left(0.7\sqrt{L^2 + T^2}\right) \]

where

\( P_n(l) \) is the compressive capacity of a lacing bar and implicitly considers the bar’s properties.

The 0.7 factor accounts for the stabilizing effect of one lace on the other.
Flexural Capacity

For compact cross-section, with $\lambda < \lambda_p$

$$M_p = Z f_y$$

For a non-compact cross-section, with $\lambda_p \leq \lambda \leq \lambda_r$

$$M_n = M_p - \left( M_p - M_r \left( \frac{\lambda - \lambda_p}{\lambda_r - \lambda_p} \right) \right)$$

where $\lambda$ is based on the controlling element, and where

$M_r = S f_y$

where

$Z$ = the plastic modulus

$S$ = the section modulus

Rivet hole creates a net section in the angle

The components of the cross-sections of built-up and laced members that are subjected to tensile stress should satisfy net section requirements if the members are to reliably develop the plastic moment. Otherwise, use the elastic capacity.
### Compactness Levels

<table>
<thead>
<tr>
<th>Width-to-Thick. Ratio</th>
<th>Term</th>
<th>Ductility</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; $\lambda_r$</td>
<td>Slender</td>
<td>None; local buckling occurs before yield</td>
</tr>
<tr>
<td>$\leq \lambda_r$</td>
<td>Non-compact</td>
<td>Little; yields before local buckling, but section won’t reach the plastic moment</td>
</tr>
<tr>
<td>$\leq \lambda_p$</td>
<td>Compact</td>
<td>Modest; section will achieve a rotational ductility of 4</td>
</tr>
<tr>
<td>$\leq \lambda_a$</td>
<td>Ductile</td>
<td>Significant; section will achieve a rotational ductility of about 8 to 10</td>
</tr>
</tbody>
</table>

### Damage Levels

- **No damage**
  - Loads don’t exceed the nominal capacity, per code

- **Minimal damage**
  - Essentially elastic performance, characterized by
    - Minor inelastic response
    - No apparent permanent deformations
    - Inconsequential yielding of secondary members
Damage Levels

- **Repairable damage**
  - Can be repaired with a minimum risk of losing functionality, i.e., without closing the bridge, characterized by
    - Yield of members, although replacement should not be necessary
    - Small permanent offsets, not interfering with functionality

- **Significant damage**
  - Minimal risk of collapse, but may require closure to repair, characterized by
    - Yield of members, possibly requiring replacement
    - Permanent offset of the structure

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Flexural Ductility – Definitions

\[ R = \frac{\theta_h}{\theta_p} \]

where
\[ \theta_h = \text{the rotation capacity} \]
\[ \theta_p = \text{the plastic rotation, from} \]
\[ \theta_p = \frac{M_p}{M_y} \theta_y \]

where
\[ M_p = \text{the plastic moment} \]
\[ M_y = \text{the yield moment, and} \ \theta_y, \text{the yield rotation is} \]
Flexural Ductility – Definitions

\[ \theta_y = \frac{M_p}{E I} \]

where

- \( E \) = the elastic modulus
- \( I \) = the moment of inertia
- \( L_p \) = the plastic hinge length, say, 10% of the distance from the point of maximum moment to the point of contraflexure.

Flexural Ductility – New Members

<table>
<thead>
<tr>
<th>Damage Level</th>
<th>( \lambda \leq \lambda_s )</th>
<th>( \lambda \leq \lambda_p )</th>
<th>( \lambda \leq \lambda_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant</td>
<td>8</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Repairable</td>
<td>4</td>
<td>2.1</td>
<td>1</td>
</tr>
<tr>
<td>Minimal</td>
<td>2</td>
<td>1.4</td>
<td>1</td>
</tr>
<tr>
<td>No</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Assumes that the axial force \( P < 0.5 A_g F_y \)

For \( 0.5 A_g F_y < P < 1.0 A_g F_y \) linearly interpolate between tabulated values and 1.0
Flexural Ductility – Laced Members

- Based on only a few tests of the cyclic behavior of laced members
- May also be based on finite element analysis
- Not to exceed values for new members, based on width-to-thickness ratios
- Reduce capacity with axial force by the same rule as for new members
- Check shear on laces
- Check net section of components

<table>
<thead>
<tr>
<th>Damage Level</th>
<th>Ductility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant</td>
<td>2</td>
</tr>
<tr>
<td>Repairable</td>
<td>1.6</td>
</tr>
<tr>
<td>Minimal</td>
<td>1.3</td>
</tr>
<tr>
<td>No</td>
<td>1</td>
</tr>
</tbody>
</table>

Flexural Ductility – Laced Member Tests

- Tested specimens with various b/t and Kt/r ratios
Flexural Ductility – Laced Member Tests

- Uang, C.M. & Kleiser, M., “Cyclic Performance of Latticed Members for San Francisco-Oakland Bay Bridge,” Report No. SSRP-97/01, Division of Structural Engineering, University of California at San Diego
- Tested members for seismic retrofit of the SFOBB

- Tested members for seismic retrofit of the SFOBB
Flexural Ductility – Perforated Members

- Based on one test of the cyclic behavior of perforated members
- May also be based on inelastic finite element analysis
- Not to exceed the values for new members, based on width-to-thickness ratios
- Reduce capacity with axial force by the same rule as for new members
- Check net section of components

<table>
<thead>
<tr>
<th>Damage Level</th>
<th>Ductility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant</td>
<td>3</td>
</tr>
<tr>
<td>Repairable</td>
<td>2.1</td>
</tr>
<tr>
<td>Minimal</td>
<td>1.4</td>
</tr>
<tr>
<td>No</td>
<td>1</td>
</tr>
</tbody>
</table>

Flexural Ductility – Perf. Member Tests

- Tested members for seismic retrofit of the SFOBB
Axial Ductility – New Members

- Values assume a compact section, i.e., $\lambda \leq \lambda_p$
- Values for significant damage are based on Dowrick, “Earthquake Resistant Design,” 2nd edition, John Wiley & Sons
- Values are for X or Z-braces / V or K-braces

\[
\text{Member} \frac{KI}{r} \sqrt{\frac{f_y}{36 \text{ ksi}}}
\]

<table>
<thead>
<tr>
<th>Damage Level</th>
<th>&lt; 40</th>
<th>&lt; 80</th>
<th>&lt; 120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant</td>
<td>4.5 / 3.0</td>
<td>3.5 / 1.8</td>
<td>2.4 / 1.2</td>
</tr>
<tr>
<td>Repairable</td>
<td>2.7 / 2.1</td>
<td>2.3 / 1.5</td>
<td>1.8 / 1.1</td>
</tr>
<tr>
<td>Minimal</td>
<td>1.7 / 1.4</td>
<td>1.5 / 1.2</td>
<td>1.3 / 1.1</td>
</tr>
<tr>
<td>No</td>
<td>1.0 / 1.0</td>
<td>1.0 / 1.0</td>
<td>1.0 / 1.0</td>
</tr>
</tbody>
</table>

Axial Ductility – Laced and Perf. Members

- Based on
  - Lee, K., & Bruneau, M.
  - Uang, C.M. & Kleiser, M.
  - Dietrich, A. & Itani, A.
- Not to exceed the values for new members based on controlling slenderness ratio of the member

<table>
<thead>
<tr>
<th>Damage Level</th>
<th>Laced</th>
<th>Perforated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Repairable</td>
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<td>2.1</td>
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<td>1.4</td>
</tr>
<tr>
<td>No</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Riveted Connections

\[ \phi \cdot R_n = \phi \cdot F_r \cdot mA_r \]

where
\[ \phi = 0.80 \]
\[ F_r = \text{the ultimate strength of the rivet in single shear} \]
\[ m = \text{the number of shear planes} \]
\[ A = \text{the nominal area of the rivet} \]

- Could also use the yield strength of rivets for higher performance and less damage
- Check bearing strength per the AASHTO LRFD

Bolted and Welded Connections

- **Bolted Connections**
  - Design per the AASHTO LRFD
  - If member ductility demands exceed unity, then connections should be 25% stronger than the members they connect

- **Welded Connections**
  - Design per the AASHTO LRFD
  - If member ductility demands exceed unity, then connections should be 25% stronger than the members they connect
  - Partial penetration welds
    - Don’t use in regions subjected to inelastic deformation
    - Elsewhere, provide 150% of the required strength, but not less than 75% of the member strength
Gusset Plates

- Shall be designed to $F_y$ for combined force and moment
- Use Thornton’s methodology to calculate forces and moments
- Shall be designed to $F_u/\sqrt{3}$ for uniform shear and to $0.74 \times F_u/\sqrt{3}$ for flexural shear
- Buckling shall be investigated using a truss analogy; with $K=0.65$
Gusset Plates

Unsupported edges should satisfy

\[ \frac{l}{t} \leq 1.6 \sqrt{\frac{E}{f_y}} \]

where

- \( l \) = the unsupported length of the edge
- \( t \) = the thickness of the gusset plate

Otherwise, add a stiffener.
Gusset Plates

- Added stiffeners should satisfy

\[
I_s \geq \max\left(9.2, 1.83 \sqrt{\frac{(b/t)^2}{2} - 144}\right) t^4
\]

where

- \( I_s \) = the moment of inertia of the stiffener (about its own axis)
- \( b \) = the tributary width of gusset plate (1/2 the edge length)
- \( t \) = the thickness of the gusset plate

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Gusset Plates

- Should be 25% stronger than the members they connect, if member ductility demands exceed unity
Fracture of the Net Section

\[ \frac{A_e}{A_g} \geq \frac{1.2 \alpha D}{A_g f_u} \]

where
- \( A_e \) = the effective net area
- \( A_g \) = the gross area of the member
- \( f_u \) = the minimum tensile strength of the member
- \( \alpha \) = the fraction of member force transferred across the net section
- \( D = \min(A_g f_y, F_{eq}, F_{max}) \)
- \( f_y \) = the yield strength of the member
- \( F_{eq} \) = the elastic seismic force, and \( F_{max} \) = the maximum force possible

Displacement Capacity of Systems

- **Portal Frames**
- **Sway Frames**
- **Support Towers**
Sway Frame

Support Towers
Pushover Analysis

Modeling Requirements

- **Column**
  - Plastic hinging element (with yield moment dependent on axial force)
  - Include axial forces acting on columns

- **Braces**
  - Phenomenological model
  - Physical model
  - Finite element model
    - Utilizes plastic hinging elements
    - Requires geometrically nonlinear analysis
Phenomenological Model

Ikeda, Mahin, & Dermitzakis

Physical Model

Ikeda & Mahin
Finite Element Model

Plastic-hinging element

Beam element

Force-Displacement Response

"Failure" of first plastic hinge

Formation of plastic hinge in leg or bucking of brace
Force-Displacement Response

Member Versus System Ductility
Pushover Analysis

- A concentrated load at the top of the structure
- A uniform load
- A uniform acceleration, or mass proportional load
- A concentrated displacement at the top of the structure
- A displacement pattern derived from the displaced shape corresponding to the peak drift of the
- A displacement pattern equal to one or more relevant mode shapes

Capacity/Demand Analysis

\[
\frac{C}{D} = \frac{\text{Member or System Capacity}}{\text{Member or System Demand}}
\]
**Force Capacity/Demand Analysis**

\[
\frac{C}{D_T} = \frac{T}{D} = \frac{\text{Tensile Strength}}{\text{Calculated Demand}}
\]

\[
\frac{C}{D_C} = \frac{C}{D} = \frac{\text{Compressive Strength}}{\text{Calculated Demand}}
\]

\[
\frac{C}{D_S} = \frac{S}{D} = \frac{\text{Connection and/or Splice Strength}}{\text{Calculated Demand}}
\]

\[
\frac{C}{D_N} = \frac{N}{D} = \frac{\text{Net Section Strength}}{\text{Calculated Demand}}
\]

**Displacement Capacity/Demand Analysis**

\[
\frac{C}{D} = \frac{\text{Member Deformation Ductility Capacity}}{\text{Member Deformation Ductility Demand}}
\]

\[
\frac{C}{D} = \frac{\text{Subsystem Displacement Capacity}}{\text{Subsystem Displacement Demand}}
\]
Threshold Values of C/D Ratio

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>Member Type</th>
<th>Main, Non-Red.</th>
<th>Main, Redundant</th>
<th>Sec., Bracing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yielding</td>
<td></td>
<td>1.0</td>
<td>0.8</td>
<td>0.67</td>
</tr>
<tr>
<td>Fracture</td>
<td></td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Buckling</td>
<td></td>
<td>1.0</td>
<td>0.8</td>
<td>0.67</td>
</tr>
<tr>
<td>Conn. or Splice</td>
<td></td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Force Capacity/Demand < 1 ?**

- **If only slightly less**
  - A minor amount of yielding is probably acceptable
  - Capacity calculation is likely to be conservative
  - These ideas are reflected in the threshold values

- **If significantly less**
  - Inelastic analysis warranted
  - Get ductility demands of the member
  - Evaluate force redistribution to other members
Case 1: $C/D_T < 1$

- The tensile capacity is less than the demand
  - Inelastic response is implied
  - Fracture of the net section should be avoided
  - Connections and splices should be 25% stronger than the member

- Retrofit it may not be required if the ratio is above the threshold value, i.e., if $C/D_T > \gamma_T$

Case 2: $C/D_T < \gamma_T$

- The tensile capacity is less than the (threshold) demand

- Retrofit should be pursued to increase the tensile strength of the member

- Reevaluate strengthened member
  - Net section
  - Splices and connections
Case 3: \( \frac{C}{D_C} < \gamma_C \) and \( \gamma_T < \frac{C}{D_T} \)

- The compressive capacity is less than the (threshold) demand
- The tensile capacity exceeds the (threshold) demand
- Retrofit should be pursued to increase the compressive strength of the member
- Reevaluate strengthened member
  - Net section
  - Splices and connections

Case 4: \( \gamma_C < \frac{C}{D_C} \) and \( \gamma_T < \frac{C}{D_T} \)

- The compressive capacity exceeds the (threshold) demand
- The tensile capacity exceeds the (threshold) demand
- Evaluate connections and splices
Case 4a: $C/D_S < \gamma_S$

- The connection or splice capacity is less than the (threshold) demand
- Strengthen connection

Case 4b: $\gamma_S < C/D_S$

- The connection or splice capacity exceeds the (threshold) demand
- No retrofit is required, if the tensile capacity exceeds the demand, i.e., $1 < C/D_T$
- Otherwise ($C/D_T < 1$), the member response is inelastic
  - Case 1 applies
  - Connections and splices should be 25% stronger than the member
Interaction of Axial Force and Flexure

**AASHTO LRFD interaction equations**

\[
\frac{P_u}{2.0P_r} + \left( \frac{M_{rx}}{M_{rx}} + \frac{M_{ry}}{M_{ry}} \right) \leq 1.0 \text{ if } \frac{P_u}{P_r} < 0.2
\]

\[
\frac{P_u}{P_r} + \frac{8}{9} \left( \frac{M_{rx}}{M_{rx}} + \frac{M_{ry}}{M_{ry}} \right) \leq 1.0 \text{ if } \frac{P_u}{P_r} \geq 0.2
\]

---

**AASHTO Inverted**

\[
2 \left( \frac{P_r}{P_u} \right) \left( \frac{M_{rx}M_{ry}}{M_{rx}M_{ry} + M_{ux}M_{ux} + M_{uy}M_{uy}} \right) \geq 1.0 \text{ if } \frac{P_r}{P_u} > 5
\]

\[
2 \left( \frac{P_r}{P_u} \right) + \left( \frac{M_{rx}M_{ry}}{M_{rx}M_{ry} + M_{ux}M_{ux} + M_{uy}M_{uy}} \right) \geq 1.0 \text{ if } \frac{P_r}{P_u} \leq 5
\]
Capacity Demand Ratio

\[
\frac{C}{D} = \begin{cases} 
2 \left( \frac{P_r}{P_u} \right) \left( \frac{M_{rz} M_{ry}}{M_{ux} M_{ry} + M_{uy} M_{rx}} \right) & \text{if } \frac{P_r}{P_u} > 5 \\
2 \left( \frac{P_r}{P_u} \right) + \left( \frac{M_{rz} M_{ry}}{M_{ux} M_{ry} + M_{uy} M_{rx}} \right) & \text{if } \frac{P_r}{P_u} \leq 5
\end{cases}
\]