Benchmark bridge No. 2 is a straight, 3-span, steel plate-girder structure with single column piers and seat-type abutments. The spans are continuous over the piers with span lengths of 105 ft, 152.5 ft, and 105 ft for a total length of 362.5 ft (Figure 2.1). The girders are spaced 11.25 ft apart with 3.75 ft overhangs for a total width of 30 ft. The built-up girders are composed of 1.625 in by 22.5 in top and bottom flange plates and 0.9375 in. by 65 in. web plate. The reinforced concrete deck slab is 8.125 in thick with 1.875 in. haunch. The support and intermediate cross-frames are of V-type configuration as shown in Figure 2.2. Cross-frame spacing is about 15 ft throughout the bridge length. The total weight of superstructure is 1,651 kips.

All the piers are single concrete columns with a diameter of 48 in, longitudinal steel ratio of 1%, and transverse steel ratio of 1%. The calculated plastic moment is equal to 3,078 kft and the plastic shear (in single curvature) is 128k. The total height of the superstructure is 24 ft above the ground. The clear height of the column is 19 ft.

The design of an isolation system for this bridge is given in this section, assuming the bridge is located on a rock site where the PGA = 0.4, $S_3 = 0.75$ and $S_1 = 0.20$. A 2-column format is used for this design example, in which the left hand column lays out a step-by-step design procedure and the right hand column applies this procedure to this particular bridge.

In addition, the design of six variations of this bridge is also provided. These seven design examples (the benchmark bridge plus six variations) are summarized in the Table 2.1.

### Table 2.1 List of Design Examples Related to Benchmark Bridge No. 2

<table>
<thead>
<tr>
<th>ID</th>
<th>Description</th>
<th>$S_1$</th>
<th>Site Class</th>
<th>Column height</th>
<th>Skew</th>
<th>Isolator type</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>Benchmark bridge</td>
<td>0.2g</td>
<td>B</td>
<td>Both 19ft, clear</td>
<td>0</td>
<td>Lead-rubber bearing</td>
</tr>
<tr>
<td>2.1</td>
<td>Change site class</td>
<td>0.2g</td>
<td>D</td>
<td>Both 19ft, clear</td>
<td>0</td>
<td>Lead rubber bearing</td>
</tr>
<tr>
<td>2.2</td>
<td>Change spectral acceleration, $S_1$</td>
<td>0.6g</td>
<td>B</td>
<td>Both 19ft, clear</td>
<td>0</td>
<td>Lead rubber bearing</td>
</tr>
<tr>
<td>2.3</td>
<td>Change isolator to FPS</td>
<td>0.2g</td>
<td>B</td>
<td>Both 19ft, clear</td>
<td>0</td>
<td>Friction pendulum</td>
</tr>
<tr>
<td>2.4</td>
<td>Change isolator to EQS</td>
<td>0.2g</td>
<td>B</td>
<td>Both 19ft, clear</td>
<td>0</td>
<td>Eradiquake</td>
</tr>
<tr>
<td>2.5</td>
<td>Change column height</td>
<td>0.2g</td>
<td>B</td>
<td>19 and 38 ft, clear</td>
<td>0</td>
<td>Lead rubber bearing</td>
</tr>
<tr>
<td>2.6</td>
<td>Change angle of skew</td>
<td>0.2g</td>
<td>B</td>
<td>Both 19ft, clear</td>
<td>45°</td>
<td>Lead rubber bearing</td>
</tr>
</tbody>
</table>
Figure 2.1 Plan of 3-Span Benchmark Bridge No. 2.

Figure 2.2 Typical Section of Superstructure and Elevation at Pier of Benchmark Bridge No. 2.
## STEP A: BRIDGE AND SITE DATA

### A1. Bridge Properties
Determine properties of the bridge:
- number of supports, $m$
- number of girders per support, $n$
- angle of skew
- weight of superstructure including railings, curbs, barriers and other permanent loads, $W_{SS}$
- weight of piers participating with superstructure in dynamic response, $W_{pp}$
- weight of superstructure, $W_j$, at each support
- pier heights (clear dimensions)
- stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge
- column flexural yield strength (minimum value)
- allowable movement at expansion joints
- isolator type if known, otherwise ‘to be selected’

#### Example 2.0
- Number of supports, $m = 4$
  - North Abutment ($m = 1$)
  - Pier 1 ($m = 2$)
  - Pier 2 ($m = 3$)
  - South Abutment ($m = 4$)
- Number of girders per support, $n = 3$
- Angle of skew = $0^\circ$
- Number of columns per support = 1
- Weight of superstructure including permanent loads, $W_{SS} = 1651.32$ k
- Weight of superstructure at each support:
  - $W_1 = 168.48$ k
  - $W_2 = 657.18$ k
  - $W_3 = 657.18$ k
  - $W_4 = 168.48$ k
- Participating weight of piers, $W_{pp} = 256.26$ k
- Effective weight (for calculation of period), $W_{eff} = W_{SS} + W_{pp} = 1907.58$ k
- Pier heights are both 19 ft (clear)
- Stiffness of each pier in the both directions (assume fixed at footing and single curvature behavior):
  - $K_{sub,pier1} = 288.87$ k/in
  - $K_{sub,pier2} = 288.87$ k/in
- Minimum flexural yield strength of single column pier = 3,078 kft (plastic moment capacity).
- Displacement capacity of expansion joints (longitudinal) = 2.5 in for thermal and other movements
- Lead-rubber isolators

### A2. Seismic Hazard
Determine seismic hazard at site:
- acceleration coefficients
- site class and site factors
- seismic zone

Plot response spectrum.

Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of $PGA$, $S_S$ and $S_I$ are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD).

Use Art. 3.2 to obtain Site Class and corresponding Site Factors ($F_{pga}$, $F_a$ and $F_c$). These data are the same as for conventional bridges and Art 3.2 refers the designer to

#### Example 2.0
- Acceleration coefficients for bridge site are given in design statement as follows:
  - $PGA = 0.40$
  - $S_I = 0.20$
  - $S_S = 0.75$

Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec.

Table 3.10.3.1-1 LRFD gives Site Class as B.

Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors:
- $F_{pga} = 1.0$
- $F_a = 1.0$
- $F_c = 1.0$
the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD.

Seismic Zone is determined by value of $S_D1$ in accordance with provisions in Table 5-1 GSID.

Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows:

- $A_s = F_{pga} \times PGA$
- $S_{DS} = F_a \times S_s$
- $S_{DI} = F_v \times S_1$

These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.

Since $0.15 < S_{DI} \leq 0.30$, bridge is located in Seismic Zone 2.

- $A_s = F_{pga} \times PGA = 1.0(0.40) = 0.40$
- $S_{DS} = F_a \times S_s = 1.0(0.75) = 0.75$
- $S_{DI} = F_v \times S_1 = 1.0(0.20) = 0.20$

- Design Response Spectrum is as below:

A3. Performance Requirements
Determine required performance of isolated bridge during Design Earthquake (1000-yr return period).

Examples of performance that might be specified by the Owner include:

- Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake.
- Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake.
- For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements)
- Reduced substructure forces for bridges on weak soils to reduce foundation costs.

A3. Performance Requirements, Example 2.0
In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield).

To remain elastic the maximum lateral load on the pier must be less than the load to yield the column. This load is taken as the plastic moment capacity (strength) of the column (3078 kft, see above) divided by the column height (24 ft). This calculation assumes the column is acting as a simple cantilever in single curvature.

Hence load to yield column = $3078 / 24 = 128.0$ k

The maximum shear in the column must therefore be less than 128 k in order to keep the column elastic and meet the required performance criterion.
STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as $K_{isol}$ are dependent on displacement ($d$), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, $k_{eff}$ is used for the effective stiffness of an isolator unit and $K_{eff}$ is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, $K_{isol}$ is used in this document in place of $k_{eff}$. There is no change in the use of $K_{eff}$ and $K_{eff,j}$, but $K_{sub}$ is used in place of $k_{sub}$.

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement ($d_{isol}$ in above figure, replaced by $d$ below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find $d$, characteristic strength, $Q_{d}$, and post elastic stiffness, $K_{d}$, for each isolator ‘j’ such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties

To begin the iterative solution, an estimate is required of:

1. Structure displacement, $d$. One way to make this estimate is to assume the effective isolation period, $T_{eff}$, is 1.0 second, take the viscous damping ratio, $\xi$, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, $B_{L}$, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)

   $d = \frac{9.79 S_{D1} T_{eff}}{B_{L}} \geq 10 S_{D1}$  

2. Characteristic strength, $Q_{d}$. This strength needs to be high enough that yield does not

   $d \geq 10 S_{D1} = 10(0.20) \geq 2.0$ in

---

<table>
<thead>
<tr>
<th>Art C7.1 GSID</th>
<th>$d = \frac{9.79 S_{D1} T_{eff}}{B_{L}} \geq 10 S_{D1}$ (B-1)</th>
<th>B1.1 Initial System Displacement and Properties, Example 2.0</th>
<th>$d \geq 10 S_{D1} = 10(0.20) \geq 2.0$ in</th>
</tr>
</thead>
</table>

---

<table>
<thead>
<tr>
<th>Isolator Displacement, $d$</th>
<th>Isolator Force, $F$</th>
<th>$d_{isol}$ = Isolator displacement</th>
<th>$d_{y}$ = Isolator yield displacement</th>
<th>$F_{y}$ = Isolator yield force</th>
<th>$d_{sub}$ = Isolator displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{y}$</td>
<td>$F_{isol}$</td>
<td>$K_{d}$ = Post-elastic stiffness of isolator</td>
<td>$K_{isol}$ = Effective stiffness of isolator</td>
<td>$K_{sub}$ = Loading and unloading stiffness (elastic stiffness)</td>
<td>$Q_{d}$ = Characteristic strength of isolator</td>
</tr>
</tbody>
</table>
occur under non-seismic loads (e.g. wind) but low enough that yield will occur during an earthquake. Experience has shown that taking $Q_d$ to be 5% of the bridge weight is a good starting point, i.e. $Q_d = 0.05W$ (B-2)

(3) Post-yield stiffness, $K_d$
Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3.

$$K_{d,min} \geq \frac{0.025W}{d}$$ (B-3)

Experience has shown that a good starting point is to take $K_d$ equal to twice this minimum value, i.e. $K_d = 0.05W/d$

B1.2 Initial Isolator Properties at Supports
Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support ‘j’ by distributing the total calculated strength, $Q_d$, and stiffness, $K_d$, values in proportion to the dead load applied at that support:

$$Q_{d,j} = Q_d \frac{W_j}{W}$$ (B-4)

and

$$K_{d,j} = K_d \frac{W_j}{W}$$ (B-5)

B1.3 Effective Stiffness of Combined Pier and Isolator System
Calculate the effective stiffness, $K_{eff,j}$, of each support ‘j’ for all supports, taking into account the stiffness of the isolators at support ‘j’ ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below for definitions (after Fig. 7.1-1 GSID).

An expression for $K_{eff,j}$ is given in Eq.7.1-6 GSID, but a more useful formula is as follows (MCEER 2006):

$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1 + \alpha_j}$$ (B-6)

where

$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$ (B-7)

and $K_{sub,j}$ for the piers are given in Step A1. For the

<table>
<thead>
<tr>
<th>$\alpha_j$</th>
<th>$K_{eff,j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8.43 \times 10^{-4}$</td>
<td>8.42 k/in</td>
</tr>
<tr>
<td>$1.21 \times 10^{-1}$</td>
<td>16.43 k/in</td>
</tr>
<tr>
<td>$1.21 \times 10^{-1}$</td>
<td>16.43 k/in</td>
</tr>
<tr>
<td>$8.43 \times 10^{-4}$</td>
<td>4.21 k/in</td>
</tr>
</tbody>
</table>

B1.2 Initial Isolator Properties at Supports, Example 2.0

$$Q_{d,j} = Q_d \frac{W_j}{W}$$

- $Q_{d,1} = 8.42$ k
- $Q_{d,2} = 32.86$ k
- $Q_{d,3} = 32.86$ k
- $Q_{d,4} = 8.42$ k

and

$$K_{d,j} = K_d \frac{W_j}{W}$$

- $K_{d,1} = 4.21$ k/in
- $K_{d,2} = 16.43$ k/in
- $K_{d,3} = 16.43$ k/in
- $K_{d,4} = 4.21$ k/in

B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 2.0

$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$

- $\alpha_1 = 8.43 \times 10^{-4}$
- $\alpha_2 = 1.21 \times 10^{-1}$
- $\alpha_3 = 1.21 \times 10^{-1}$
- $\alpha_4 = 8.43 \times 10^{-4}$

$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1 + \alpha_j}$$

- $K_{eff,1} = 8.42$ k/in
- $K_{eff,2} = 31.09$ k/in
- $K_{eff,3} = 31.09$ k/in
- $K_{eff,4} = 8.42$ k/in
abutments, take \( K_{sub,j} \) to be a large number, say 10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high values for \( K_{sub,j} \) will give unconservative results for column moments and shear forces.

### B1.4 Total Effective Stiffness

Calculate the total effective stiffness, \( K_{eff} \), of the bridge:

\[
K_{eff} = \sum_{j=1}^{m} K_{eff,j} \tag{B-8}
\]

#### B1.4 Total Effective Stiffness, Example 2.0

\[
K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 79.02 \text{ k/in}
\]

### B1.5 Isolation System Displacement at Each Support

Calculate the displacement of the isolation system at support ‘j’, \( d_{isol,j} \), for all supports:

\[
d_{isol,j} = \frac{d}{1 + \alpha_j} \tag{B-9}
\]

#### B1.5 Isolation System Displacement at Each Support, Example 2.0

- \( d_{isol,1} = 2.00 \text{ in} \)
- \( d_{isol,2} = 1.79 \text{ in} \)
- \( d_{isol,3} = 1.79 \text{ in} \)
- \( d_{isol,4} = 2.00 \text{ in} \)

### B1.6 Isolation System Stiffness at Each Support

Calculate the stiffness of the isolation system at support ‘j’, \( K_{isol,j} \), for all supports:

\[
K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \tag{B-10}
\]

#### B1.6 Isolation System Stiffness at Each Support, Example 2.0

- \( K_{isol,1} = 8.43 \text{ k/in} \)
- \( K_{isol,2} = 34.84 \text{ k/in} \)
- \( K_{isol,3} = 34.84 \text{ k/in} \)
- \( K_{isol,4} = 8.43 \text{ k/in} \)
B1.7 Substructure Displacement at Each Support

Calculate the displacement of substructure \( j \), \( d_{\text{sub},j} \), for all supports:

\[
d_{\text{sub},j} = d - d_{\text{isol},j} \quad (B-11)
\]

\( d_{\text{sub},1} = 0.002 \text{ in} \)
\( d_{\text{sub},2} = 0.215 \text{ in} \)
\( d_{\text{sub},3} = 0.215 \text{ in} \)
\( d_{\text{sub},4} = 0.002 \text{ in} \)

---

B1.8 Lateral Load in Each Substructure

Calculate the lateral load in substructure \( j \), \( F_{\text{sub},j} \), for all supports:

\[
F_{\text{sub},j} = K_{\text{sub},j}d_{\text{sub},j} \quad (B-12)
\]

where values for \( K_{\text{sub},j} \) are given in Step A1.

\( F_{\text{sub},1} = 16.84 \text{ k} \)
\( F_{\text{sub},2} = 62.18 \text{ k} \)
\( F_{\text{sub},3} = 62.18 \text{ k} \)
\( F_{\text{sub},4} = 16.84 \text{ k} \)

---

B1.9 Column Shear Force at Each Support

Calculate the shear force in column \( k \) at support \( j \), \( F_{\text{col},j,k} \), assuming equal distribution of shear for all columns at support \( j \):

\[
F_{\text{col},j,k} = \frac{F_{\text{sub},j}}{\text{# of columns at support } j} \quad (B-13)
\]

Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.

\( F_{\text{col},2,1} = 62.18 \text{ k} \)
\( F_{\text{col},3,1} = 62.18 \text{ k} \)

These column shears are less than the plastic shear capacity of each column (128k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.

---

B1.10 Effective Period and Damping Ratio

Calculate the effective period, \( T_{\text{eff}} \), and the viscous damping ratio, \( \xi \), of the bridge:

\[
T_{\text{eff}} = 2\pi \frac{W_{\text{eff}}}{gK_{\text{eff}}} \quad (B-14)
\]

and

\[
\xi = \frac{2}{\pi} \sum (Q_{a,j}(d_{\text{isol},j} - d_{\text{y},j})) (B-15)
\]

where \( d_{y,j} \) is the yield displacement of the isolator. For friction-based isolators, \( d_{y,j} = 0 \). For other types of isolators \( d_{y,j} \) is usually small compared to \( d_{\text{isol},j} \) and has negligible effect on \( \xi \). Hence it is suggested that for the Simplified Method, set \( d_{y,j} = 0 \) for all isolator types. See Step B2.2 where the value of \( d_{y,j} \) is revisited.

\[
T_{\text{eff}} = 2\pi \frac{1907.58}{386.4(79.02)} = 1.57 \text{ sec}
\]

and taking \( d_{y,j} = 0 \):

\[
\xi = \frac{2}{\pi} \sum (Q_{a,j}(d_{\text{isol},j} - 0)) (B-15) = 0.30
\]
for the Multimode Spectral Analysis Method.

<table>
<thead>
<tr>
<th><strong>B1.11 Damping Factor</strong></th>
<th><strong>B1.11 Damping Factor, Example 2.0</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate the damping factor, $B_L$, and the displacement, $d$, of the bridge:</td>
<td>Since $\xi = 0.30 \geq 0.3$</td>
</tr>
<tr>
<td>Eq. 7.1-3 $B_L = \left{ \begin{array}{ll} \frac{\xi}{0.05}^{0.3}, &amp; \xi &lt; 0.3 \ 1.7, &amp; \xi \geq 0.3 \end{array} \right.$ (B-16)</td>
<td>$B_L = 1.70$</td>
</tr>
<tr>
<td>GSID</td>
<td>and</td>
</tr>
<tr>
<td>Eq. 7.1-4 $d = \frac{9.79S_{D1}T_{eff}}{B_L}$ (B-17)</td>
<td>$d = \frac{9.79S_{D1}T_{eff}}{B_L} = \frac{9.79(0.2)1.57}{1.70} = 1.81 \text{ in}$</td>
</tr>
<tr>
<td>GSID</td>
<td><strong>B1.12 Convergence Check, Example 2.0</strong></td>
</tr>
<tr>
<td>Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement.</td>
<td>Since the calculated value for displacement, $d (=1.81)$ is not close to that assumed at the beginning of the cycle (Step B1.1, $d = 2.0$), use the value of 1.81 as the new assumed displacement and repeat from Step B1.3.</td>
</tr>
<tr>
<td>This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5).</td>
<td>After three iterations, convergence is reached at a superstructure displacement of 1.65 in, with an effective period of 1.43 seconds, and a damping factor of 1.7 (30% damping ratio). The displacement in the isolators at Pier 1 is 1.44 in and the effective stiffness of the same isolators is 42.78 k/in.</td>
</tr>
<tr>
<td>After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust $Q_d$ and $K_d$ (Step B1.1) and repeat. It may take several attempts to find the right combination of $Q_d$ and $K_d$. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both.</td>
<td>See spreadsheet in Table B1.12-1 for results of final iteration.</td>
</tr>
<tr>
<td>Note that Art 9 GSID requires that a minimum clearance be provided equal to $8 S_{D1} T_{eff} / B_L$. (B-18)</td>
<td>Ignoring the weight of the hammerhead, the column shear force must equal the isolator shear force for equilibrium. Hence column shear $= 42.78 (1.44) = 61.60 \text{ k}$ which is less than the maximum allowable (128 k) if elastic behavior is to be achieved (as required in Step A3).</td>
</tr>
<tr>
<td></td>
<td>Also the superstructure displacement $= 1.65 \text{ in}$, which is less than the available clearance of 2.5 in.</td>
</tr>
<tr>
<td></td>
<td>Therefore the above solution is acceptable and go to Step B2.</td>
</tr>
<tr>
<td></td>
<td>Note that available clearance (2.5 in) is greater than minimum required which is given by:</td>
</tr>
<tr>
<td></td>
<td>$\frac{8 S_{D1} T_{eff}}{B_L} = \frac{8(0.20)1.43}{1.7} = 1.35 \text{ in}$</td>
</tr>
</tbody>
</table>
### Table B1.12-1 Simplified Method Solution for Design Example 2.0 – Final Iteration

**SIMPLIFIED METHOD SOLUTION**

- **Step A1,A2**
  - $W_{ss}$  
  - $W_{sp}$  
  - $W_{eff}$  
  - $S_{d1}$  
  - $n$
  - 1651.32  
  - 256.26  
  - 1907.58  
  - 0.2  
  - 3

- **Step B1.1**
  - $d$  
  - 1.65  
  - Assumed displacement
  - $Q_d$  
  - 82.57  
  - Characteristic strength
  - $K_d$  
  - 50.04  
  - Post-yield stiffness

- **Step**
  - A1  
  - B1.2  
  - B1.2  
  - A1  
  - B1.3  
  - B1.3  
  - B1.5  
  - B1.6  
  - B1.7  
  - B1.8  
  - B1.10  
  - B1.10

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<th>B1.2</th>
<th>A1</th>
<th>B1.3</th>
<th>B1.3</th>
<th>B1.5</th>
<th>B1.6</th>
<th>B1.7</th>
<th>B1.8</th>
<th>B1.10</th>
<th>B1.10</th>
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<tr>
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<td>8.424</td>
<td>5.105</td>
<td>10,000.00</td>
<td>0.001022</td>
<td>10.206</td>
<td>1.648</td>
<td>10.216</td>
<td>0.002</td>
<td>16.839</td>
<td>13.885</td>
<td>27.785</td>
</tr>
<tr>
<td>Q_d,j</td>
<td>10.206</td>
<td>1.648</td>
<td>10.216</td>
<td>0.002</td>
<td>16.839</td>
<td>13.885</td>
<td>27.785</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>K_d,j</td>
<td>10.206</td>
<td>1.648</td>
<td>10.216</td>
<td>0.002</td>
<td>16.839</td>
<td>13.885</td>
<td>27.785</td>
<td></td>
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<td></td>
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<tr>
<td>K_eff,j</td>
<td>10.206</td>
<td>1.648</td>
<td>10.216</td>
<td>0.002</td>
<td>16.839</td>
<td>13.885</td>
<td>27.785</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K_sub,j</td>
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<td>1.648</td>
<td>10.216</td>
<td>0.002</td>
<td>16.839</td>
<td>13.885</td>
<td>27.785</td>
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<td></td>
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</tr>
<tr>
<td>d_isol,j</td>
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<td>1.648</td>
<td>10.216</td>
<td>0.002</td>
<td>16.839</td>
<td>13.885</td>
<td>27.785</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>d_sub,j</td>
<td>10.206</td>
<td>1.648</td>
<td>10.216</td>
<td>0.002</td>
<td>16.839</td>
<td>13.885</td>
<td>27.785</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K_eff,j(d_isol,j + d_sub,j)^2</td>
<td>10.206</td>
<td>1.648</td>
<td>10.216</td>
<td>0.002</td>
<td>16.839</td>
<td>13.885</td>
<td>27.785</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Step B1.10**
  - $T_{eff}$  
  - 1.43  
  - Effective period
  - $\xi$  
  - 0.30  
  - Equivalent viscous damping ratio

- **Step B1.11**
  - $B_l$  
  - 1.71  
  - (B-15)
  - $B_l$  
  - 1.70  
  - Damping Factor

<table>
<thead>
<tr>
<th>Step</th>
<th>B2.1</th>
<th>B2.1</th>
<th>B2.3</th>
<th>B2.6</th>
<th>B2.8</th>
</tr>
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<tbody>
<tr>
<td>$Q_{d,l}$</td>
<td>2.808</td>
<td>1.702</td>
<td>3.405</td>
<td>1.69</td>
<td>3.363</td>
</tr>
<tr>
<td>$K_{d,l}$</td>
<td>10.953</td>
<td>6.638</td>
<td>14.259</td>
<td>1.20</td>
<td>15.766</td>
</tr>
<tr>
<td>$K_{sub,l}$</td>
<td>10.953</td>
<td>6.638</td>
<td>14.259</td>
<td>1.20</td>
<td>15.766</td>
</tr>
<tr>
<td>$d_{sub,l}$</td>
<td>2.808</td>
<td>1.702</td>
<td>3.405</td>
<td>1.69</td>
<td>3.363</td>
</tr>
</tbody>
</table>
B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.

Once convergence has been achieved, obtain the following:
- longitudinal and transverse displacements \( (u_t, v_t) \) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

### B2.1 Characteristic Strength

Calculate the characteristic strength, \( Q_{d,i} \), and post-elastic stiffness, \( K_{d,i} \), of each isolator ‘i’ as follows:

\[
Q_{d,i} = \frac{Q_{d,j}}{n} \quad \text{(B-19)}
\]

and

\[
K_{d,i} = \frac{K_{d,j}}{n} \quad \text{(B-20)}
\]

where values for \( Q_{d,j} \) and \( K_{d,j} \) are obtained from the final cycle of iteration in the Simplified Method (Step B1).

### B2.1 Characteristic Strength, Example 2.0

Dividing the results for \( Q_d \) and \( K_d \) in Step B1.12 (see Table B1.12-1) by the number of isolators at each support \((n = 3)\), the following values for \( Q_d/\text{isolator} \) and \( K_d/\text{isolator} \) are obtained:

- \( Q_{d,1} = 8.42/3 = 2.81 \text{ k} \)
- \( Q_{d,2} = 32.86/3 = 10.95 \text{ k} \)
- \( Q_{d,3} = 32.86/3 = 10.95 \text{ k} \)
- \( Q_{d,4} = 8.42/3 = 2.81 \text{ k} \)

and

- \( K_{d,1} = 5.10/3 = 1.70 \text{ k/in} \)
- \( K_{d,2} = 19.92/3 = 6.64 \text{ k/in} \)
- \( K_{d,3} = 19.92/3 = 6.64 \text{ k/in} \)
- \( K_{d,4} = 5.10/3 = 1.70 \text{ k/in} \)

Note that the \( K_d \) values per support used above are from the final iteration given in Table B1.12-1. These are not the same as the initial values in Step B1.2, because they have been adjusted from cycle to cycle, such that the total \( K_d \) summed over all the isolators satisfies the minimum lateral restoring force requirement for the bridge, i.e. \( K_{d,\text{total}} = 0.05 \text{ W/d} \). See Step B1.1. Since \( d \) varies from cycle to cycle, \( K_d \) varies from cycle to cycle.

### B2.2 Initial Stiffness and Yield Displacement

Calculate the initial stiffness, \( K_{u,i} \), and the yield displacement, \( d_{y,i} \), for each isolator ‘i’ as follows:

1. For friction-based isolators \( K_{u,i} = \infty \) and \( d_{y,i} = 0 \).
2. For other types of isolators, and in the absence of isolator-specific information, take

\[
K_{u,i} = 10K_{d,i} \quad \text{(B-21)}
\]

and then

\[
d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \quad \text{(B-22)}
\]

### B2.2 Initial Stiffness and Yield Displacement, Example 2.0

Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate \( K_{u,i} \) and \( d_{y,i} \) for an isolator on Pier 1 as follows:

\[
K_{u,1} = 10K_{d,1} = 10(6.64) = 66.4 \text{ k/in}
\]

and

\[
d_{y,1} = \frac{Q_{d,1}}{K_{u,1} - K_{d,1}} = \frac{10.95}{(66.4 - 6.64)} = 0.18 \text{ in}
\]

As expected, the yield displacement is small compared to the expected isolator displacement (~2...
in) and will have little effect on the damping ratio (Eq B-15). Therefore take \( d_{yi} = 0 \).

**B2.3 Isolator Effective Stiffness, \( K_{isol} \)**

Calculate the isolator stiffness, \( K_{isol,i} \), of each isolator ‘i’:

\[
K_{isol,i} = \frac{K_{isol,i}}{n} \quad (B-23)
\]

**B2.3 Isolator Effective Stiffness, \( K_{isol} \), Example 2.0**

Dividing the results for \( K_{isol} \) (Step B1.12) among the 3 isolators at each support, the following values for \( K_{isol} \) /isolator are obtained:

- \( K_{isol,1} = \frac{10.22}{3} = 3.41 \text{ k/in} \)
- \( K_{isol,2} = \frac{42.78}{3} = 14.26 \text{ k/in} \)
- \( K_{isol,3} = \frac{42.78}{3} = 14.26 \text{ k/in} \)
- \( K_{isol,4} = \frac{10.22}{3} = 3.41 \text{ k/in} \)

**B2.4 Three-Dimensional Bridge Model**

Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (\( K_x \) and \( K_y \) in global coordinates, \( K_2 \) and \( K_3 \) in typical local coordinates) is the \( K_{isol} \) value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single ‘stick’ provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator. If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam, a more rigorous model is recommended.

**B2.4 Three-Dimensional Bridge Model, Example 2.0**

Although the bridge in this Design Example is regular and is without skew or curvature, a 3-dimensional finite element model was developed for this Step, as shown below.

**B2.5 Composite Design Response Spectrum**

Modify the response spectrum obtained in Step A2 to obtain a ‘composite’ response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (\( \xi \)) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, \( T_{eff} \), by the damping factor, \( B_L \).

**B2.5 Composite Design Response Spectrum, Example 2.0**

From the final results of Simplified Method (Step B1.12), \( B_L = 1.70 \) and \( T_{eff} = 1.43 \) sec. Hence the transition in the composite spectrum from 5% to 30% damping occurs at 0.8 \( T_{eff} = 0.8 \times 1.43 = 1.14 \) sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods \( \geq 1.14 \) sec by 1.70.
B2.6 Multimodal Analysis of Finite Element Model
Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.

B2.6 Multimodal Analysis of Finite Element Model, Example 2.0
Results of modal analysis of the example bridge are summarized in Table B2.6-1. Here the modal periods and mass participation factors of the first 12 modes are given. The first three modes are the principal transverse, longitudinal, and torsion modes with periods of 1.60, 1.46, and 1.39 sec respectively. The period of the longitudinal mode (1.46 sec) is very close to that calculated in the Simplified Method. The mass participation factors indicate there is no coupling between these three modes (probably due to the symmetric nature of the bridge) and the high values for the first and second modes (92% and 94% respectively) indicate the bridge is responding essentially in a single mode of vibration in each direction. Similar results to that obtained by the Simplified Method are therefore expected.

<table>
<thead>
<tr>
<th>Mode No</th>
<th>Period Sec</th>
<th>UX</th>
<th>UY</th>
<th>Mass Participation Ratios</th>
</tr>
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<tbody>
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<td>1</td>
<td>1.604</td>
<td>0.000</td>
<td>0.919</td>
<td>0.000 0.952 0.000 0.697</td>
</tr>
<tr>
<td>2</td>
<td>1.463</td>
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<td>0.000</td>
<td>0.000 0.000 0.000 0.231</td>
</tr>
<tr>
<td>3</td>
<td>1.394</td>
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<td>0.003</td>
<td>0.000 0.013 0.000 0.002</td>
</tr>
<tr>
<td>4</td>
<td>0.727</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000 0.000 0.000 0.000</td>
</tr>
<tr>
<td>5</td>
<td>0.346</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000 0.000 0.000 0.000</td>
</tr>
<tr>
<td>6</td>
<td>0.345</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000 0.010 0.000 0.000</td>
</tr>
<tr>
<td>7</td>
<td>0.279</td>
<td>0.000</td>
<td>0.003</td>
<td>0.000 0.013 0.000 0.002</td>
</tr>
<tr>
<td>8</td>
<td>0.268</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000 0.000 0.000 0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.267</td>
<td>0.058</td>
<td>0.000</td>
<td>0.000 0.000 0.000 0.000</td>
</tr>
<tr>
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<td>0.000</td>
<td>0.000 0.000 0.129 0.000</td>
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<td>11</td>
<td>0.183</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000 0.000 0.000 0.001</td>
</tr>
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</table>

Table B2.6-1 Modal Properties of Bridge Example 2.0 – First Iteration
Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- \( d_{isol,1} = 1.69 \ (1.65) \) in
- \( d_{isol,2} = 1.20 \ (1.44) \) in
- \( d_{isol,3} = 1.20 \ (1.44) \) in
- \( d_{isol,4} = 1.69 \ (1.65) \) in

B2.7 Convergence Check
Compare the resulting displacements at the superstructure level (\( \delta \)) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 2.0
The results for isolator displacements are close but not close enough (15% difference at the piers)

Go to Step B2.8 and update properties for a second cycle of iteration.
### B2.8 Update $K_{isol,i}$, $K_{eff,j}$, $\xi$ and $B_L$

Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows:

\[ K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_d \]  \hspace{1cm} (B-24)

Recalculate $K_{eff,j}$:

\[ K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \]  \hspace{1cm} (B-25)

Recalculate system damping ratio, $\xi$:

\[ \xi = \frac{2 \sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i}))}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j}))^2} \]  \hspace{1cm} (B-26)

Recalculate system damping factor, $B_L$:

\[ B_L = \begin{cases} \left(\frac{\xi}{0.05}\right)^{0.3} & \text{if } \xi \leq 0.3 \\ \left(\frac{\xi}{1.7}\right) & \text{if } \xi > 0.3 \end{cases} \]  \hspace{1cm} (B-27)

Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.

### B2.8 Update $K_{isol,i}$, $K_{eff,j}$, $\xi$ and $B_L$, Example 2.0

Updated values for $K_{isol,i}$ are given below (previous values are in parentheses):

- $K_{isol,1} = 3.36$ (3.41) k/in
- $K_{isol,2} = 15.77$ (14.26) k/in
- $K_{isol,3} = 15.77$ (14.26) k/in
- $K_{isol,4} = 3.36$ (3.41) k/in

Since the isolator displacements are relatively close to previous results no significant change in the damping ratio is expected. Hence $K_{eff,j}$ and $\xi$ are not recalculated and $B_L$ is taken at 1.70.

### B2.6 Multimodal Analysis Second Iteration, Example 2.0

Reanalysis gives the following values for the isolator displacements (numbers in parentheses are those from the previous cycle):

- $d_{isol,1} = 1.66$ (1.69) in
- $d_{isol,2} = 1.15$ (1.20) in
- $d_{isol,3} = 1.15$ (1.20) in
- $d_{isol,4} = 1.66$ (1.69) in

Since the change in effective period is very small (1.43 to 1.46 sec) and no change has been made to $B_L$, there is no need to construct a new composite response spectrum in this case. Go back to Step B2.6 (see immediately below).

### B2.7 Convergence Check, Example 2.0

Satisfactory agreement has been reached on this second cycle. Go to Step B2.9.

### B2.9 Superstructure and Isolator Displacements, Example 2.0

From the above analysis:

- superstructure displacements in the longitudinal ($x_L$) and transverse ($y_L$) directions of the bridge are:
  - $x_L = 1.69$ in

---

<table>
<thead>
<tr>
<th>B2.8 Update $K_{isol,i}$, $K_{eff,j}$, $\xi$ and $B_L$</th>
<th>B2.8 Update $K_{isol,i}$, $K_{eff,j}$, $\xi$ and $B_L$, Example 2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows: $K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_d$</td>
<td>Updated values for $K_{isol,i}$ are given below (previous values are in parentheses):</td>
</tr>
<tr>
<td>$K_{isol,1} = 3.36$ (3.41) k/in</td>
<td>$K_{isol,1} = 3.36$ (3.41) k/in</td>
</tr>
<tr>
<td>$K_{isol,2} = 15.77$ (14.26) k/in</td>
<td>$K_{isol,2} = 15.77$ (14.26) k/in</td>
</tr>
<tr>
<td>$K_{isol,3} = 15.77$ (14.26) k/in</td>
<td>$K_{isol,3} = 15.77$ (14.26) k/in</td>
</tr>
<tr>
<td>$K_{isol,4} = 3.36$ (3.41) k/in</td>
<td>$K_{isol,4} = 3.36$ (3.41) k/in</td>
</tr>
<tr>
<td>Since the isolator displacements are relatively close to previous results no significant change in the damping ratio is expected. Hence $K_{eff,j}$ and $\xi$ are not recalculated and $B_L$ is taken at 1.70.</td>
<td></td>
</tr>
<tr>
<td>Since the change in effective period is very small (1.43 to 1.46 sec) and no change has been made to $B_L$, there is no need to construct a new composite response spectrum in this case. Go back to Step B2.6 (see immediately below).</td>
<td></td>
</tr>
<tr>
<td><strong>B2.6 Multimodal Analysis Second Iteration, Example 2.0</strong></td>
<td><strong>B2.6 Multimodal Analysis Second Iteration, Example 2.0</strong></td>
</tr>
<tr>
<td>Reanalysis gives the following values for the isolator displacements (numbers in parentheses are those from the previous cycle):</td>
<td>Reanalysis gives the following values for the isolator displacements (numbers in parentheses are those from the previous cycle):</td>
</tr>
<tr>
<td>$d_{isol,1} = 1.66$ (1.69) in</td>
<td>$d_{isol,1} = 1.66$ (1.69) in</td>
</tr>
<tr>
<td>$d_{isol,2} = 1.15$ (1.20) in</td>
<td>$d_{isol,2} = 1.15$ (1.20) in</td>
</tr>
<tr>
<td>$d_{isol,3} = 1.15$ (1.20) in</td>
<td>$d_{isol,3} = 1.15$ (1.20) in</td>
</tr>
<tr>
<td>$d_{isol,4} = 1.66$ (1.69) in</td>
<td>$d_{isol,4} = 1.66$ (1.69) in</td>
</tr>
<tr>
<td>Go to Step B2.6</td>
<td>Go to Step B2.6</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>B2.7 Convergence Check</th>
<th>B2.7 Convergence Check, Example 2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compare results and determine if convergence has been reached. If so go to Step B2.9. Otherwise Go to Step B2.8.</td>
<td>Satisfactory agreement has been reached on this second cycle. Go to Step B2.9</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>B2.9 Superstructure and Isolator Displacements</th>
<th>B2.9 Superstructure and Isolator Displacements, Example 2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Once convergence has been reached, obtain</td>
<td>From the above analysis:</td>
</tr>
<tr>
<td>$\circ$ superstructure displacements in the longitudinal ($x_L$) and transverse ($y_L$) directions of the bridge, and</td>
<td>$\circ$ superstructure displacements in the longitudinal ($x_L$) and transverse ($y_L$) directions are:</td>
</tr>
<tr>
<td>$\circ$ isolator displacements in the longitudinal ($u_L$) and transverse ($v_L$) directions of the bridge, for</td>
<td>$x_L = 1.69$ in</td>
</tr>
</tbody>
</table>
for this load case (i.e., longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.

\[ y_L = 0.0 \text{ in} \]

- isolator displacements in the longitudinal \((u_L)\) and transverse \((v_L)\) directions are:
  - Abutments: \(u_L = 1.66\) in, \(v_L = 0.00\) in
  - Piers: \(u_L = 1.15\) in, \(v_L = 0.00\) in

All isolators at same support have the same displacements.

### B2.10 Pier Bending Moments and Shear Forces

Obtain the pier bending moments and shear forces in the longitudinal \((M_{PLL}, V_{PLL})\) and transverse \((M_{PTL}, V_{PTL})\) directions at the critical locations for the longitudinally-applied seismic loading.

### B2.10 Pier Bending Moments and Shear Forces, Example 2.0

Bending moments in single column pier in the longitudinal \((M_{PLL})\) and transverse \((M_{PTL})\) directions are:

- \(M_{PLL} = 0\)
- \(M_{PTL} = 1602\) kft

Shear forces in single column pier in the longitudinal \((V_{PLL})\) and transverse \((V_{PTL})\) directions are:

- \(V_{PLL} = 67.16\) k
- \(V_{PTL} = 0\)

### B2.11 Isolator Shear and Axial Forces

Obtain the isolator shear \((V_{LL}, V_{TL})\) and axial forces \((P_L)\) for the longitudinally-applied seismic loading.

### B2.11 Isolator Shear and Axial Forces, Example 2.0

Isolator shear and axial forces are summarized in Table B2.11-1.

#### Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

<table>
<thead>
<tr>
<th>Substructure</th>
<th>Isolator</th>
<th>(V_{LL}) (k)</th>
<th>(V_{TL}) (k)</th>
<th>(P_L) (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abutment</td>
<td>1</td>
<td>5.63</td>
<td>0</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.63</td>
<td>0</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.63</td>
<td>0</td>
<td>1.29</td>
</tr>
<tr>
<td>Pier</td>
<td>1</td>
<td>18.19</td>
<td>0</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>18.25</td>
<td>0</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>18.19</td>
<td>0</td>
<td>0.77</td>
</tr>
</tbody>
</table>

The difference between the longitudinal shear force in the column \((V_{PLL} = 67.16\)k\) and the sum of the isolator shear forces at the same Pier \((54.63\) k\) is about 12.5 k. This is due to the inertia force developed in the hammerhead cap beam which weighs about 128 k and can generate significant additional demand on the column (about a 23% increase in this case).
STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements \((u_T, v_T)\) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal \((u_T)\) and transverse \((v_T)\) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 2.0

Key results from repeating Steps B1 and B2 (Simplified and Multimode Spectral Methods) are:

- \(T_{ef} = 1.52\) sec
- Superstructure displacements in the longitudinal \((x_T)\) and transverse \((y_T)\) directions are as follows: \(x_T = 0\) and \(y_T = 1.75\) in
- Isolator displacements in the longitudinal \((u_T)\) and transverse \((v_T)\) directions as follows:
  - Abutments \(u_T = 0.00\) in, \(v_T = 1.75\) in
  - Piers \(u_T = 0.00\) in, \(v_T = 0.71\) in
- Pier bending moments in the longitudinal \((M_{PLT})\) and transverse \((M_{PTT})\) directions are as follows:
  - \(M_{PLT} = 1548.33\) kft and \(M_{PTT} = 0\)
- Pier shear forces in the longitudinal \((V_{PLT})\) and transverse \((V_{PTT})\) directions are as follows:
  - \(V_{PLT} = 0\) and \(V_{PTT} = 60.75\) k
- Isolator shear and axial forces are in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

<table>
<thead>
<tr>
<th>Substructure</th>
<th>Isolator</th>
<th>(V_{LT}) (k) Long. shear due to transv. EQ</th>
<th>(V_{TT}) (k) Transv. shear due to transv. EQ</th>
<th>(P_T) (k) Axial forces due to transv. EQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abutment</td>
<td>1</td>
<td>0.0</td>
<td>5.82</td>
<td>13.51</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0</td>
<td>5.83</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0</td>
<td>5.82</td>
<td>13.51</td>
</tr>
<tr>
<td>Pier</td>
<td>1</td>
<td>0.0</td>
<td>15.40</td>
<td>26.40</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0</td>
<td>15.57</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0</td>
<td>15.40</td>
<td>26.40</td>
</tr>
</tbody>
</table>

The difference between the transverse shear force in the column \((V_{PLL} = 60.75\text{k})\) and the sum of the isolator shear forces at the same Pier \((46.37\text{k})\) is about 14.4 k. This is due to the inertia force developed in the hammerhead cap beam which weighs about 128 k and can generate significant additional demand on the column (about 31%).
**STEP D. CALCULATE DESIGN VALUES**

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

<table>
<thead>
<tr>
<th>D1. Design Isolator Displacements</th>
<th>D2. Design Moments and Shears</th>
</tr>
</thead>
<tbody>
<tr>
<td>Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, ( d_i ), for each isolator by combining the displacements from the longitudinal (( u_L ) and ( v_L )) and transverse (( u_T ) and ( v_T )) cases as follows:</td>
<td></td>
</tr>
<tr>
<td><strong>u_1 = u_L + 0.3u_T</strong></td>
<td><strong>d_1 = \max(R_1, R_2)</strong></td>
</tr>
<tr>
<td><strong>v_1 = v_L + 0.3v_T</strong></td>
<td></td>
</tr>
<tr>
<td><strong>R_1 = \sqrt{u_1^2 + v_1^2}</strong></td>
<td></td>
</tr>
<tr>
<td><strong>u_2 = 0.3u_L + u_T</strong></td>
<td></td>
</tr>
<tr>
<td><strong>v_2 = 0.3v_L + v_T</strong></td>
<td></td>
</tr>
<tr>
<td><strong>R_2 = \sqrt{u_2^2 + v_2^2}</strong></td>
<td></td>
</tr>
<tr>
<td><strong>d_1 = \max(R_1, R_2)</strong></td>
<td></td>
</tr>
</tbody>
</table>

**D1. Design Isolator Displacements at Pier 1, Example 2.0**
To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

**Load Case 1:**
\[ u_1 = u_L + 0.3u_T = 1.0(1.15) + 0.3(0) = 1.15 \text{ in} \]
\[ v_1 = v_L + 0.3v_T = 1.0(0) + 0.3(0.71) = 0.21 \text{ in} \]
\[ R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{1.15^2 + 0.21^2} = 1.17 \text{ in} \]

**Load Case 2:**
\[ u_2 = 0.3u_L + u_T = 0.3(1.15) + 1.0(0) = 0.35 \text{ in} \]
\[ v_2 = 0.3v_L + v_T = 0.3(0) + 1.0(0.71) = 0.71 \text{ in} \]
\[ R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{0.35^2 + 0.71^2} = 0.79 \text{ in} \]

**Governing Case:**
Total design displacement, \( d_1 = \max(R_1, R_2) \) = 1.17 in

**D2. Design Moments and Shears in Pier 1, Example 2.0**
Design moments and shear forces are calculated for Pier 1 below, to illustrate the process.

**Load Case 1:**
\[ V_{PLL} = V_{PLL} + 0.3V_{PLT} = 1.0(67.16) + 0.3(0) = 67.16 \text{ k} \]
\[ V_{PTL} = V_{PTL} + 0.3V_{PTT} = 1.0(0) + 0.3(60.75) = 18.23 \text{ k} \]
\[ R_1 = \sqrt{V_{L1}^2 + V_{T1}^2} = \sqrt{67.16^2 + 18.23^2} = 69.59 \text{ k} \]

**Load Case 2:**
\[ V_{PLL} = 0.3V_{PLL} + V_{PLT} = 0.3(67.16) + 1.0(0) = 20.15 \text{ k} \]
\[ V_{PTL} = 0.3V_{PTL} + V_{PTT} = 0.3(0) + 1.0(60.75) = 60.75 \text{ k} \]
\[ R_2 = \sqrt{V_{L2}^2 + V_{T2}^2} = \sqrt{20.15^2 + 60.75^2} = 64.00 \text{ k} \]

**Governing Case:**
Design column shear = \( \max(R_1, R_2) \) = 69.59 k
STEP E. DESIGN OF LEAD-RUBBER (ELASTOMERIC) ISOLATORS

A lead-rubber isolator is an elastomeric bearing with a lead core inserted on its vertical centreline. When the bearing and lead core are deformed in shear, the elastic stiffness of the lead provides the initial stiffness \((K_u)\). With increasing lateral load the lead yields almost perfectly plastically, and the post-yield stiffness \(K_d\) is given by the rubber alone. More details are given in MCEER 2006.

While both circular and rectangular bearings are commercially available, circular bearings are more commonly used. Consequently the procedure given below focuses on circular bearings. The same steps can be followed for rectangular bearings, but some modifications will be necessary.

When sizing the physical dimensions of the bearing, plan dimensions \((B, d_L)\) should be rounded up to the next \(\frac{1}{4}\)“ increment, while the total thickness of elastomer, \(T_r\), is specified in multiples of the layer thickness. Typical layer thicknesses for bearings with lead cores are \(\frac{1}{4}\)“ and \(\frac{3}{8}\)“.

High quality natural rubber should be specified for the elastomer. It should have a shear modulus in the range 60-120 psi and an ultimate elongation-at-break in excess of 5.5. Details can be found in rubber handbooks or in MCEER 2006.

The following design procedure assumes the isolators are bolted to the masonry and sole plates. Isolators that use shear-only connections (and not bolts) require additional design checks for stability which are not included below. See MCEER 2006.

**E1. Required Properties**

Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1).

- the required characteristic strength, \(Q_d\), per isolator
- the required post-elastic stiffness, \(K_d\), per isolator
- the total design displacement, \(d_t\), for each isolator, and
- the maximum applied dead and live load \((P_{DL}, P_{LL})\) and seismic load \((P_{SL})\) which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator.

**E1. Required Properties, Example 2.0**

The design of one of the exterior isolators on a pier is given below to illustrate the design process for lead-rubber isolators.

From previous work

- \(Q_d/\text{isolator} = 10.95 \text{ k}\)
- \(K_d/\text{isolator} = 6.76 \text{ k/in}\)
- Total design displacement, \(d_t = 1.17 \text{ in}\)
- \(P_{DL} = 187 \text{ k}\)
- \(P_{LL} = 123 \text{ k}\)
- \(P_{SL} = 26.4 \text{ k} \) (Table C1-1)

Note that the \(K_d\) value per isolator used above is from the final iteration of the analysis. It is not the same as the initial value in Step B2.1 (6.64 k/in), because it has been adjusted from cycle to cycle, such that the total \(K_d\) summed over all the isolators satisfies the minimum lateral restoring force requirement for the bridge, i.e. \(K_{d\text{total}} = 0.05 \text{ W/d}\) See Step B1.1. Since \(d\) varies from cycle to cycle, \(K_{dj}\) varies from cycle to cycle.
### E2. Isolator Sizing

#### E2.1 Lead Core Diameter

Determine the required diameter of the lead plug, \( d_L \), using:

\[
d_L = \sqrt{\frac{Q_d}{0.9}} \tag{E-1}
\]

See Step E2.5 for limitations on \( d_L \).

#### E2.1 Lead Core Diameter, Example 2.0

\[
d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{10.95}{0.9}} = 3.49 \text{ in}
\]

#### E2.2 Plan Area and Isolator Diameter

Although no limits are placed on compressive stress in the GSID, (maximum strain criteria are used instead, see Step E3) it is useful to begin the sizing process by assuming an allowable stress of, say, 1.6 ksi.

Then the bonded area of the isolator is given by:

\[
A_b = \frac{P_{DL} + P_{LL}}{1.6} \text{ in}^2 \tag{E-2}
\]

and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by:

\[
B = \sqrt{\frac{4A_b}{\pi} + d_L^2} \tag{E-3}
\]

Round the bonded diameter, \( B \), to nearest quarter inch, and recalculate actual bonded area using

\[
A_b = \frac{\pi}{4}(B^2 - d_L^2) \tag{E-4}
\]

Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually 1/2 inch each side). In this case the overall diameter, \( B_o \), is given by:

\[
B_o = B + 1.0 \tag{E-5}
\]

#### E2.2 Plan Area and Isolator Diameter, Example 2.0

\[
A_b = \frac{P_{DL} + P_{LL}}{1.6} \text{ in}^2 = \frac{187 + 123}{1.6} = 193.75 \text{ in}^2
\]

\[
B = \sqrt{\frac{4A_b}{\pi} + d_L^2} = \sqrt{\frac{4(193.75)}{\pi} + 3.49^2} = 16.09 \text{ in}
\]

Round \( B \) up to 16.25 in and the actual bonded area is:

\[
A_b = \frac{\pi}{4}(16.25^2 - 3.49^2) = 197.84 \text{ in}^2
\]

\[
B_o = 16.25 + 2(0.5) = 17.25 \text{ in}
\]

#### E2.3 Elastomer Thickness and Number of Layers

Since the shear stiffness of the elastomeric bearing is given by:

\[
K_d = \frac{GA_b}{T_r} \tag{E-6}
\]

where \( G \) = shear modulus of the rubber, and \( T_r \) = the total thickness of elastomer, it follows Eq. E-6 may be used to obtain \( T_r \) given a required value for \( K_d \)

\[
T_r = \frac{GA_b}{K_d} \tag{E-7}
\]

A typical range for shear modulus, \( G \), is 60-120 psi. Higher and lower values are available and are used in special applications.

#### E2.3 Elastomer Thickness and Number of Layers, Example 2.0

Select \( G \), shear modulus of rubber, = 100 psi (0.1 ksi)

Then

\[
T_r = \frac{GA_b}{K_d} = \frac{0.1(197.84)}{6.76} = 2.93 \text{ in}
\]
If the layer thickness is \( t_r \), the number of layers, \( n \), is given by:

\[
n = \frac{T_r}{t_r}
\]

rounded up to the nearest integer.

Note that because of rounding the plan dimensions and the number of layers, the actual stiffness, \( K_d \), will not be exactly as required. Reanalysis may be necessary if the differences are large.

**E2.4 Overall Height**

The overall height of the isolator, \( H \), is given by:

\[
H = n \cdot t_r + (n - 1) \cdot t_e + 2 \cdot t_e
\]

where \( t_r \) = thickness of an internal shim (usually about 1/8 in), and \( t_e \) = combined thickness of end cover plate (0.5 in) and outer plate (1.0 in).

**E2.4 Overall Height, Example 2.0**

\[
H = 12(0.25) + 11(0.125) + 2*1.5 = 7.375 \text{ in}
\]

**E2.5 Lead Core Size Check**

Experience has shown that for optimum performance of the lead core it must not be too small or too large. The recommended range for the diameter is as follows:

\[
\frac{B}{3} \geq d_L \geq \frac{B}{6}
\]

**E2.5 Lead Core Size Check, Example 2.0**

Since \( B = 16.25 \) check

\[
\frac{16.25}{3} \geq d_L \geq \frac{16.25}{6}
\]

i.e., \( 5.41 \geq d_L \geq 2.71 \)

Since \( d_L = 3.49 \), lead core size is acceptable.

**E3. Strain Limit Check**

Art. 14.2 and 14.3 GSID requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e.,

\[
\gamma_c + \gamma_{r,eq} + 0.5\gamma_r \leq 5.5
\]

where \( \gamma_c \), \( \gamma_{r,eq} \), and \( \gamma_r \) are defined below.

(a) \( \gamma_c \) is the maximum shear strain in the layer due to compression and is given by:

\[
\gamma_c = \frac{D_c \cdot \sigma_s}{G S}
\]

where \( D_c \) is shape coefficient for compression in circular bearings = 1.0, \( \sigma_s = P_{DL}/A_b \), \( G \) is shear modulus, and \( S \) is the layer shape factor given by:

\[
S = \frac{A_b}{\pi B t_r}
\]

(b) \( \gamma_{r,eq} \) is the shear strain due to earthquake loads and is given by:

\[
\gamma_{r,eq} = \frac{d_t}{T_r}
\]
(c) $\gamma_r$ is the shear strain due to rotation and is given by:

$$\gamma_r = \frac{D_r B^2 \theta}{t_r T_r}$$  \hspace{1cm} (E-15)

where $D_r$ is shape coefficient for rotation in circular bearings = 0.375, and $\theta$ is design rotation due to DL, LL and construction effects. Actual value for $\theta$ may not be known at this time and a value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Art. 14.4.2.1).

Substitution in Eq E-11 gives

$$\gamma_c + 0.5\gamma_r + 0.5(1.32) = 1.66 \leq 5.5 \text{ OK}$$

#### E4. Vertical Load Stability Check

Art 12.3 GSID requires the vertical load capacity of all isolators be at least 3 times the applied vertical loads (DL and LL) in the laterally undeformed state.

Further, the isolation system shall be stable under 1.2(DL+SL) at a horizontal displacement equal to either

- 2 x total design displacement, $d_t$, if in Seismic Zone 1 or 2,
- or
- 1.5 x total design displacement, $d_t$, if in Seismic Zone 3 or 4.

#### E4. Vertical Load Stability Check, Example 2.0

E4.1 Vertical Load Stability in Undeformed State

The critical load capacity of an elastomeric isolator at zero shear displacement is given by

$$P_{cr(\Delta=0)} = \frac{K_d H_{eff}}{2} \left[ \sqrt{1 + \frac{4\pi^2 K_\theta}{K_d H_{eff}^2}} - 1 \right]$$  \hspace{1cm} (E-16)

where

- $H_{eff} = T_r + T_s$
- $T_s$ = total shim thickness
- $K_\theta = E_b l / T_r$
- $E_b = E(1 + 0.67S^2)$
- $E$ = elastic modulus of elastomer = 3G
- $l = \pi B^4 / 64$

It is noted that typical elastomeric isolators have high shape factors, $S$, in which case:

$$\frac{4\pi^2 K_\theta}{K_d H_{eff}^2} \gg 1$$  \hspace{1cm} (E-17)

and Eq. E-16 reduces to:

$$P_{cr(\Delta=0)} = \pi \sqrt{K_d K_\theta}$$  \hspace{1cm} (E-18)

Check that:

$$E = 3G = 3(0.1) = 0.3 \text{ ksi}$$

$$E_b = 0.3(1 + 0.67(15.50^2)) = 48.38 \text{ ksi}$$

$$l = \pi \frac{16.25^4}{64} = 3,422.8 \text{ in}^4$$

$$K_\theta = \frac{48.38(3,422.8)}{3.0} = 55,201 \text{ kin/rad}$$

$$K_d = \frac{G A_b}{T_r} = \frac{0.1(197.84)}{3.0} = 6.59 \text{ k/in}$$

$$P_{cr(\Delta=0)} = \pi \sqrt{6.59(55,201)} = 1895.5 \text{ k}$$
### E4.2 Vertical Load Stability in Deformed State

The critical load capacity of an elastomeric isolator at shear displacement $\Delta$ may be approximated by:

$$\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} \geq 3 \quad (E-19)$$

Evaluating:

$$\frac{1895.5}{(187 + 123)} = 6.11 \geq 3 \quad OK$$

where

$$P_{cr(\Delta)} = \frac{A_r}{A_{gross}} P_{cr(\Delta=0)} \quad (E-20)$$

\[
\begin{align*}
A_r &= B^2 (\delta - \sin \delta) / 4 \\
\delta &= 2\cos^{-1} (\Delta / B) \\
A_{gross} &= \pi B^2 / 4
\end{align*}
\]

It follows that:

$$\frac{A_r}{A_{gross}} = \frac{(\delta - \sin \delta)}{\pi} \quad (E-21)$$

Check that:

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} \geq 1 \quad (E-22)$$

### E5. Design Review

The basic dimensions of the isolator designed above are as follows:

- 17.25 in (od) x 7.375 in (high) x 3.49 in dia. lead core
- Volume, excluding steel end and cover plates: 1,022 in$^3$

Although this design satisfies all the required criteria, the vertical load stability ratios (Eq. E-19 and E-22) are much higher than required (6.11 vs 3.0) and total rubber shear strain (1.66) is much less than the maximum allowable (5.5), as shown in Step E3. In other words, the isolator is not working very hard and a redesign appears to be indicated to obtain a smaller isolator with more optimal properties (as well as less cost).

This redesign is outlined below. It begins by increasing the allowable compressive stress from 1.6 to 3.2 ksi to obtain initial sizes. Remember that no
limits are placed on compressive stress in GSID, only a limit on strain.

E2.1

\[ d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{10.95}{0.9}} = 3.49 \text{ in} \]

E2.2

\[ A_b = \frac{P_{DL} + P_{LL}}{3.2} \text{ in}^2 = \frac{187 + 123}{3.2} = 96.87 \text{ in}^2 \]

\[ B = \sqrt{\frac{4 A_b}{\pi} + d_L^2} = \sqrt{\frac{4 \cdot (96.87)}{\pi} + 3.49^2} = 11.64 \]

Round \( B \) up to 12.5 in and the actual bonded area becomes:

\[ A_b = \frac{\pi}{4} (12.5^2 - 3.49^2) = 113.16 \text{ in}^2 \]

\[ B_a = 12.5 + 2(0.5) = 13.5 \text{ in} \]

E2.3

\[ T_e = \frac{G A_b}{K_d} = 0.1(113.16) = 1.67 \text{ in} \]

\[ n = \frac{1.67}{0.25} = 6.7 \]

Round up to nearest integer, i.e. \( n = 7 \).

E2.4

\[ H = 7(0.25) + 6(0.125) + 2 \cdot 1.5 = 5.5 \text{ in} \]

E2.5

Since \( B = 12.5 \) check

\[ \frac{12.5}{3} \geq d_L \geq \frac{12.5}{6} \]

i.e., \( 4.17 \geq d_L \geq 2.08 \)

Since \( d_L = 3.49 \), size of lead core is acceptable.

E3.

\[ \sigma_s = \frac{187.0}{113.16} = 1.652 \text{ ksi} \]

\[ S = \frac{113.16}{\pi 12.5(0.25)} = 11.53 \]

\[ \gamma_c = \frac{1.0(1.652)}{0.1(11.53)} = 1.43 \]

\[ \gamma_{s,eq} = \frac{1.17}{1.75} = 0.67 \]
\[ \gamma_r = \frac{0.375(12.5^2)(0.01)}{0.25(1.75)} = 1.34 \]

\[ \gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 1.43 + 0.67 + 0.5(1.34) = 2.77 \leq 5.5 \text{ OK} \]

**E4.1**

\[ E = 3G = 3(0.1) = 0.3 \text{ ksi} \]

\[ E_b = 0.3(1 + 0.67(11.53^2)) = 26.89 \text{ ksi} \]

\[ I = \frac{12.5^4}{64} = 1,198.4 \text{ in}^4 \]

\[ K_p = \frac{26.89(1198.4)}{1.75} = 18,411.9 \text{ kN/m} \]

\[ K_d = \frac{GA_b}{T} = \frac{0.1(113.16)}{1.75} = 6.47 \text{ kN/m} \]

\[ P_{cr(\Delta=0)} = \pi \sqrt{6.47(18411.9)} = 1084.0 \text{ kN} \]

\[ \frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} = \frac{1084.0}{(187 + 123)} = 3.50 \geq 3 \text{ OK} \]

**E4.2**

\[ \delta = 2\cos^{-1}\left(\frac{2.34}{12.5}\right) = 2.765 \]

\[ \frac{A_r}{A_{gross}} = \left(\frac{2.76 - \sin2.76}{\pi}\right) = 0.763 \]

\[ P_{cr(\Delta)} = 0.763(1084.0) = 827.15k \]

\[ \frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{827.15}{1.2(187) + 26.4} = 3.30 \geq 1 \text{ OK} \]

**E5.**

The basic dimensions of the redesigned isolator are as follows:

13.5 in (od) x 5.5 in (high) x 3.49 in dia. lead core

and the volume, excluding steel end and cover plates, = 358 in³

This design reduces the excessive vertical stability ratio of the previous design (it is now 3.50 vs 3.0
required) and the total layer shear strain is increased (2.77 vs 5.5 max allowable). Furthermore, the isolator volume is decreased from 1,022 in³ to 358 in³. This design is clearly more efficient than the previous one.

E6. Minimum and Maximum Performance Check
Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of \(K_d\) and \(Q_d\), which are found using system property modification factors, \(\lambda\), as indicated in Table E6-1.

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

### Table E6-1. Minimum and maximum values for \(K_d\) and \(Q_d\)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Expression</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1.2-1</td>
<td>(K_{d,max} = K_d \lambda_{max,Kd})</td>
<td>(E-23)</td>
</tr>
<tr>
<td>8.1.2-2</td>
<td>(K_{d,min} = K_d \lambda_{min,Kd})</td>
<td>(E-24)</td>
</tr>
<tr>
<td>8.1.2-3</td>
<td>(Q_{d,max} = Q_d \lambda_{max,Qd})</td>
<td>(E-25)</td>
</tr>
<tr>
<td>8.1.2-4</td>
<td>(Q_{d,min} = Q_d \lambda_{min,Qd})</td>
<td>(E-26)</td>
</tr>
</tbody>
</table>

### Table E6-2. Minimum and maximum values for system property modification factors.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Expression</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.2.1-1</td>
<td>(\lambda_{min,Kd} = (\lambda_{min,t,Kd} \lambda_{min,a,Kd} \lambda_{min,v,Kd} \lambda_{min,tr,Kd} \lambda_{min,c,Kd}) \lambda_{min,scrag,Kd})</td>
<td>(E-27)</td>
</tr>
<tr>
<td>8.2.1-2</td>
<td>(\lambda_{max,Kd} = (\lambda_{max,t,Kd} \lambda_{max,a,Kd} \lambda_{max,v,Kd} \lambda_{max,tr,Kd} \lambda_{max,c,Kd}) \lambda_{max,scrag,Kd})</td>
<td>(E-28)</td>
</tr>
<tr>
<td>8.2.1-3</td>
<td>(\lambda_{min,Qd} = (\lambda_{min,t,Qd} \lambda_{min,a,Qd} \lambda_{min,v,Qd} \lambda_{min,tr,Qd} \lambda_{min,c,Qd}) \lambda_{min,scrag,Qd})</td>
<td>(E-29)</td>
</tr>
<tr>
<td>8.2.1-4</td>
<td>(\lambda_{max,Qd} = (\lambda_{max,t,Qd} \lambda_{max,a,Qd} \lambda_{max,v,Qd} \lambda_{max,tr,Qd} \lambda_{max,c,Qd}) \lambda_{max,scrag,Qd})</td>
<td>(E-30)</td>
</tr>
</tbody>
</table>

E6. Minimum and Maximum Performance Check, Example 2.0
Minimum Property Modification factors are:
\(\lambda_{min,Kd} = 1.0\)
\(\lambda_{min,Qd} = 1.0\)

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are:
\(\lambda_{max,Kd} = 1.1\)
\(\lambda_{max,Qd} = 1.1\)
\(\lambda_{max,t,Kd} = 1.1\)
\(\lambda_{max,t,Qd} = 1.4\)
\(\lambda_{max,scrag,Kd} = 1.0\)
\(\lambda_{max,scrag,Qd} = 1.0\)

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:
\(\lambda_{max,Kd} = 1.0 + 0.1(0.66) = 1.066\)
\(\lambda_{max,Qd} = 1.0 + 0.1(0.66) = 1.066\)
\(\lambda_{max,t,Kd} = 1.0 + 0.1(0.66) = 1.066\)
\(\lambda_{max,t,Qd} = 1.0 + 0.4(0.66) = 1.264\)
\(\lambda_{max,scrag,Kd} = 1.0\)
\(\lambda_{max,scrag,Qd} = 1.0\)

Therefore the maximum overall modification factors
\(\lambda_{max,Kd} = 1.066(1.066)1.0 = 1.14\)
\(\lambda_{max,Qd} = 1.066(1.264)1.0 = 1.35\)

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:
\(Q_{d,max} = 13.5 (10.95) = 14.78\) k
and
\(K_{d,max} = 1.14(6.76) = 7.71\) k/in
Adjustment factors are applied to individual $\lambda$-factors (except $\lambda_\circ$) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all $\lambda$-factors that deviate from unity but only to the portion of the $\lambda$-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:
- 1.00 for critical bridges
- 0.75 for essential bridges
- 0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,\min}$ and $Q_{d,\min}$ and again with $K_{d,\max}$ and $Q_{d,\max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,\min}$ and $Q_{d,\min}$) and maximum forces by the second case ($K_{d,\max}$ and $Q_{d,\max}$).

**E7. Design and Performance Summary**

**E7.1 Isolator dimensions**

Summarize final dimensions of isolators:
- Overall diameter (includes cover layer)
- Overall height
- Diameter lead core
- Bonded diameter
- Number of rubber layers
- Thickness of rubber layers
- Total rubber thickness
- Thickness of steel shims
- Shear modulus of elastomer

**E7.2 Bridge Performance**

Summarize bridge performance:
- Maximum superstructure displacement (longitudinal)
- Maximum superstructure displacement (transverse)
- Maximum superstructure displacement

---

**Table E7.1-1 Isolator Dimensions**

<table>
<thead>
<tr>
<th>Isolator Location</th>
<th>Overall diam. (in)</th>
<th>Overall height (in)</th>
<th>Diam. lead core (in)</th>
<th>Bonded diam (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under edge girder on Pier 1</td>
<td>13.5</td>
<td>5.5</td>
<td>3.49</td>
<td>12.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Isolator Location</th>
<th>No. of rubber layers</th>
<th>Rubber layers thickness (in)</th>
<th>Total rubber thickness (in)</th>
<th>Steel shim thickness (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under edge girder on Pier 1</td>
<td>7</td>
<td>0.25</td>
<td>1.75</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Shear modulus of elastomer = 100 psi

**E7.2 Bridge Performance, Example 2.0**

Bridge performance is summarized in Table E7.2-1 where it is seen that the maximum column shear is 71.74k. This less than the column plastic shear (128k) and therefore the required performance criterion is satisfied (fully elastic behavior). Furthermore the maximum longitudinal displacement is 1.69 in which...
- Maximum column shear (resultant)
- Maximum column moment (about transverse axis)
- Maximum column moment (about longitudinal axis)
- Maximum column torque

Check required performance as determined in Step A3, is satisfied.

Check required performance as determined in Step A3, is satisfied.

is less than the 2.5in available at the abutment expansion joints and is therefore acceptable.

**Table E7.2-1 Summary of Bridge Performance**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum superstructure</td>
<td>1.69 in</td>
</tr>
<tr>
<td>displacement (longitudinal)</td>
<td></td>
</tr>
<tr>
<td>Maximum superstructure</td>
<td>1.75 in</td>
</tr>
<tr>
<td>displacement (transverse)</td>
<td></td>
</tr>
<tr>
<td>Maximum superstructure</td>
<td>2.27 in</td>
</tr>
<tr>
<td>displacement (resultant)</td>
<td></td>
</tr>
<tr>
<td>Maximum column shear (resultant)</td>
<td>71.74 k</td>
</tr>
<tr>
<td>Maximum column moment</td>
<td>1,657 kft</td>
</tr>
<tr>
<td>about transverse axis</td>
<td></td>
</tr>
<tr>
<td>Maximum column moment</td>
<td>1,676 kft</td>
</tr>
<tr>
<td>about longitudinal axis</td>
<td></td>
</tr>
<tr>
<td>Maximum column torque</td>
<td>21.44 kft</td>
</tr>
</tbody>
</table>