Investigation of Principal Axes in a Linear MDOF Structure

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Summary

While it is well known that principal axes are nonexistent in asymmetric structures, they are usually recognized in the analysis and design of symmetric structures. However, imperfections in material properties and human errors during construction might lead to nonexistence of principal axes even if the structure is geometrically symmetric. This paper describes a research project intended to experimentally find the principal axes of a geometrically symmetric MDOF structure. Test results suggest that the experimental model indeed has no principal axes.

Introduction

Civil engineers typically accept large uncertainties in the design of structures by using a probabilistic safety factor to account for the uncertainties. However, large safety factors might not be an economical solution. Besides, probabilistic safety factors do not always lead to conservative designs. Since there are always imperfections in material properties and human errors during construction, structures cannot be symmetric with respect to any axis or plan. Consequently, the response of a structure to an external force applied in a given direction always has components in directions other than that of the external force. While this phenomenon has been realized by many engineers and researchers, proper identification of its causes has never been implemented. Most civil engineers believe that geometrically symmetric structures should have defined principal axes. This leads to the assumption that the response of the structure to a static load applied along one of the principal axes has no components in the direction perpendicular to that of the applied force. However, it has been proven mathematically that this phenomenon rarely exists in the real world even in the case of static forces. If the response in the direction perpendicular to that of the external force is small, then assuming principal axes exist might be conservative, but if the response in the perpendicular direction is large, traditional design practice needs to be revised. In the case of dynamic forces, the same phenomenon leads theoretically to an infinitely large response of undamped structures when the structure’s natural period reaches resonance with the frequency of the external force. Hence, any minor imperfection leads the structure to suffer the effects of hugely uncertain design forces. In the past, researchers concluded that asymmetric structures have no principal axes and that their response could be calculated by considering torsional effects. This paper strongly emphasizes the idea that structures have no principal axes even if they are geometrically symmetric. In addition, it is shown that the dynamic response in the direction perpendicular to that of the excitation (cross effect) increases with the level of imperfections in structure properties.
Since structures can never be perfectly symmetric in the real world, cross effect always exists. Careful experimental tests have been conducted to verify its existence and to quantify its effects. In order to differentiate between cross coupling of asymmetric buildings and cross coupling effect in symmetric buildings with some engineering-tolerated imperfections, the experimental model was designed as symmetric as possible in order to avoid torsional cross effect.

**Experimental Study**

If the non-existence of principal axes in MDOF systems is theoretically shown, it is certain then that cross effect always occurs in the real world. In most cases, inputs along the x-axis cause response along the y-axis, and this response can sometimes be quite large. In other words, if the theory holds, then conventional earthquake engineering has to be improved in order to account for the coupling effect. Hence, by carefully conducting experimental tests, the idea of nonexistence of principal axes in MDOF systems under dynamic loads will be reinforced, and experimental results can then be used as evidence of theoretical results. Tests described in this paper are part of the first experimental research aimed to prove and quantify the existence of cross effect in linear MDOF systems under dynamic loads.

A 3-story steel model was built and its response under dynamic loads was carefully examined. The structure has two axes of geometrical symmetry. Since principal axes always lay in perpendicular directions, one of them must lay within 90° from any line passing through the center of geometry (if the structure has indeed any set of principal axes). Hence, experiments were setup to change the direction of the excitation from 0° to 100°. If the structure has any principal axis, the response in the direction perpendicular to that of the excitation should then be zero when the direction of the excitation is aligned with one of the principal axes.

**Experimental Setup and Procedure**

Recognizing the importance of experimental results, the structure was carefully designed so that it has two planes of geometric symmetry. No weight was added to the structure apart from the self-weight. The design dimensions of the structure are 5 ft long and 4 ft wide. Each floor is four ft tall. All connections are welded and assumed rigidly connected. Floor diaphragms are made with four L3x2.5x1/4 angles at the perimeter and four additional L2x2x1/8 angles were welded in parallel at the center of the diaphragm. The diaphragms are assumed rigid. The foundation of the structure was clamped at all times during all experiments. The lateral stiffness of the structure was provided by four C3x5 steel channels at the four exterior frames of the structure. The channels were originally aligned in such a way that the resulting weak axis is perpendicular to the loading direction. However, the channels were also designed to be able to rotate 360° vertically at each floor to allow for maximum change of the global stiffness matrix of the structure. In order to accommodate such design criteria, the steel channels and the rigid diaphragms were not connected with welds but with 5/8” diameter high strength bolts, post-tensioned with a 125 lb-ft torque to simulate rigid connections. Portions of the steel angles were cut off to allow 360° rotation of the steel channels. However, additional 10”x10” triangular steel plates (1.5” thick) were welded to the steel angles to prevent local yielding of the diaphragm. Since the steel channels were connected to the rigid diaphragms by bolts, 4”x3.5” (1/8” thick) steel plates were welded to the channels to allow for easy fastening of the channels to the rigid diaphragms.

The structure was designed to behave linearly under maximum design ground acceleration (0.353g) along the weak axis. SAP2000 nonlinear computer analysis program was used to check design
stresses. A safety factor equal to four was used in the design of the structure to account for the unforeseen dynamic amplification of structural response caused by cross effect.

Sixteen strain-gage type accelerometers manufactured by three different companies (Sensotec, Kulite and Endeveco) were used to monitor the response of the structure. The sensitivity of all strain gages is approximately equal to 30 mv/g. However, all accelerometers were calibrated to a sensitivity equal to 1v/g so that data obtained from each sensor are all in the same equivalent unit. The accelerometers could measure acceleration only in the direction along which they were mounted.

In order to identify the principal axes of the structure, a total of 21 tests were conducted varying the direction of the excitation at 5° intervals.

**Summary of Results**

Results obtained from the experiments are presented in three-dimensional plots. Figures 1-4 show the response of the structure in the direction perpendicular to that of the input excitation at the ground, 2nd, 3rd and 4th floor, respectively. Maximum structural responses (regardless of the corresponding frequency) are summarized in Figure 5. $Y_{1_{max}}$, $Y_{2_{max}}$, $Y_{3_{max}}$ and $Y_{4_{max}}$ in Figure 5 are the maximum response of the structure in the direction perpendicular to that of the input excitation at the ground, 2nd, 3rd and the 4th floor, respectively.
Discussion of Results

Based on the results shown in Figure 5, none of the responses in the direction perpendicular to that of the ground excitation reaches zero in the 0° - 100° range considered. This indicates that the structure does not have principal axes under dynamic loads.

Conclusions

Cross effect always exists even if the structure is designed with two axes of geometric symmetry. This leads to the conclusion that principal axes do not exist in MDOF systems. Since results show promising repeatability, they are convincing, and the experiments can be considered valid. Experimental results not only prove the non-existence of principal axes in MDOF systems, but also indicate the amount of structural response amplification at the resonance frequency.

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